Nucleon Structure from Lattice QCD

Philipp Hägler

TECHNISCHE UNIVERSITÄT MÜNCHEN

supported by

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

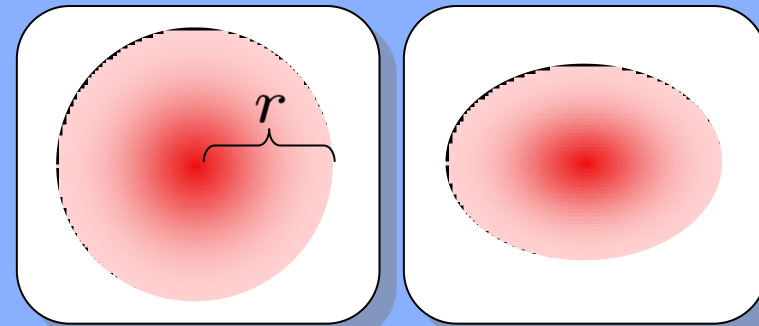
excellence cluster universe

Charges, magnetic moments, (r)ms radii,...

Form factors

$g_A, g_T, \langle r^2 \rangle, \kappa, \ldots$

$F_1(Q^2), F_2(Q^2), \ldots$
$G_A(Q^2), G_P(Q^2), \ldots$



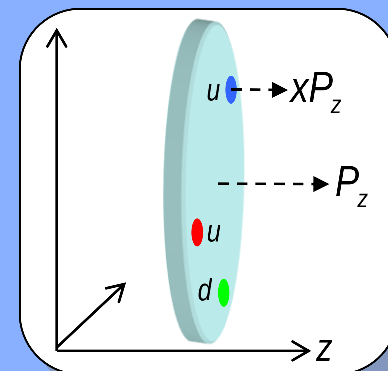
PDFs

$q(x), \Delta q(x), \delta q(x), g(x), \ldots$

Distribution in (longitudinal) momentum

q/g spin content/distribution

long. momentum fraction



GPDs

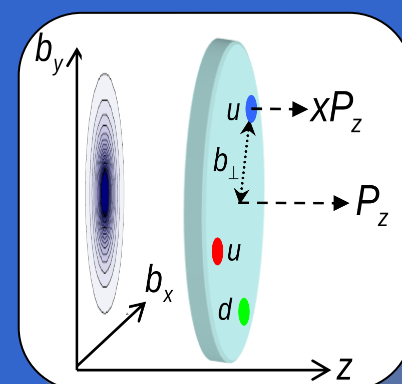
$H(x,\xi,t), E(x,\xi,t)$
$\tilde{H}(x,\xi,t), \tilde{E}(x,\xi,t)$
$H_T(x,\xi,t), E_T(x,\xi,t), \ldots$

$t=(P'-P)^2$

Distribution of momentum&spin
In coordinate space $[t] \overset{FT}{\rightarrow} [b_\perp]$

Orbital angular momentum

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

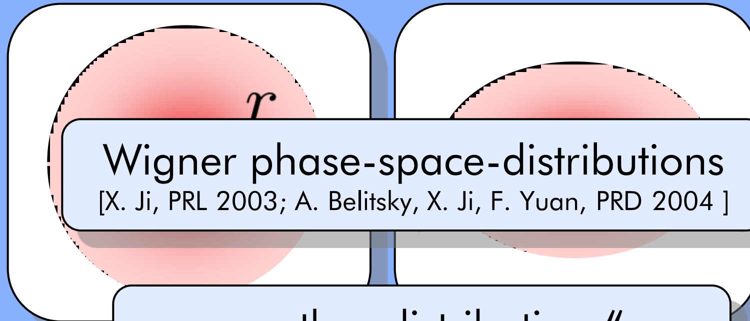


Charges, magnetic moments, (r)ms radii,...

$g_A, g_T, \langle r^2 \rangle, \kappa, \ldots$

Form factors

$F_1(Q^2), F_2(Q^2), \ldots$
$G_A(Q^2), G_P(Q^2), \ldots$



Wigner phase-space-distributions
[X. Ji, PRL 2003; A. Belitsky, X. Ji, F. Yuan, PRD 2004]

„mother-distributions“
[Meissner, Metz, Schlegel JHEP 0908:056,2009]

PDFs

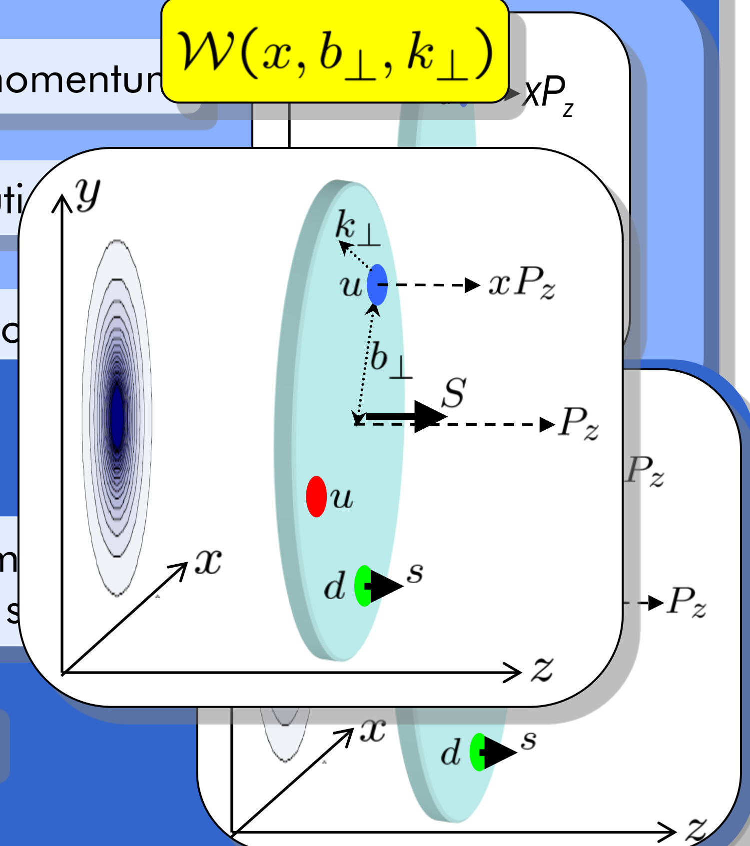
$q(x), \Delta q(x), \delta q(x), g(x), \ldots$

Distribution in (longitudinal) momentum

q/g spin content/distributi

long. momentum fractio

$\mathcal{W}(x, b_\perp, k_\perp)$



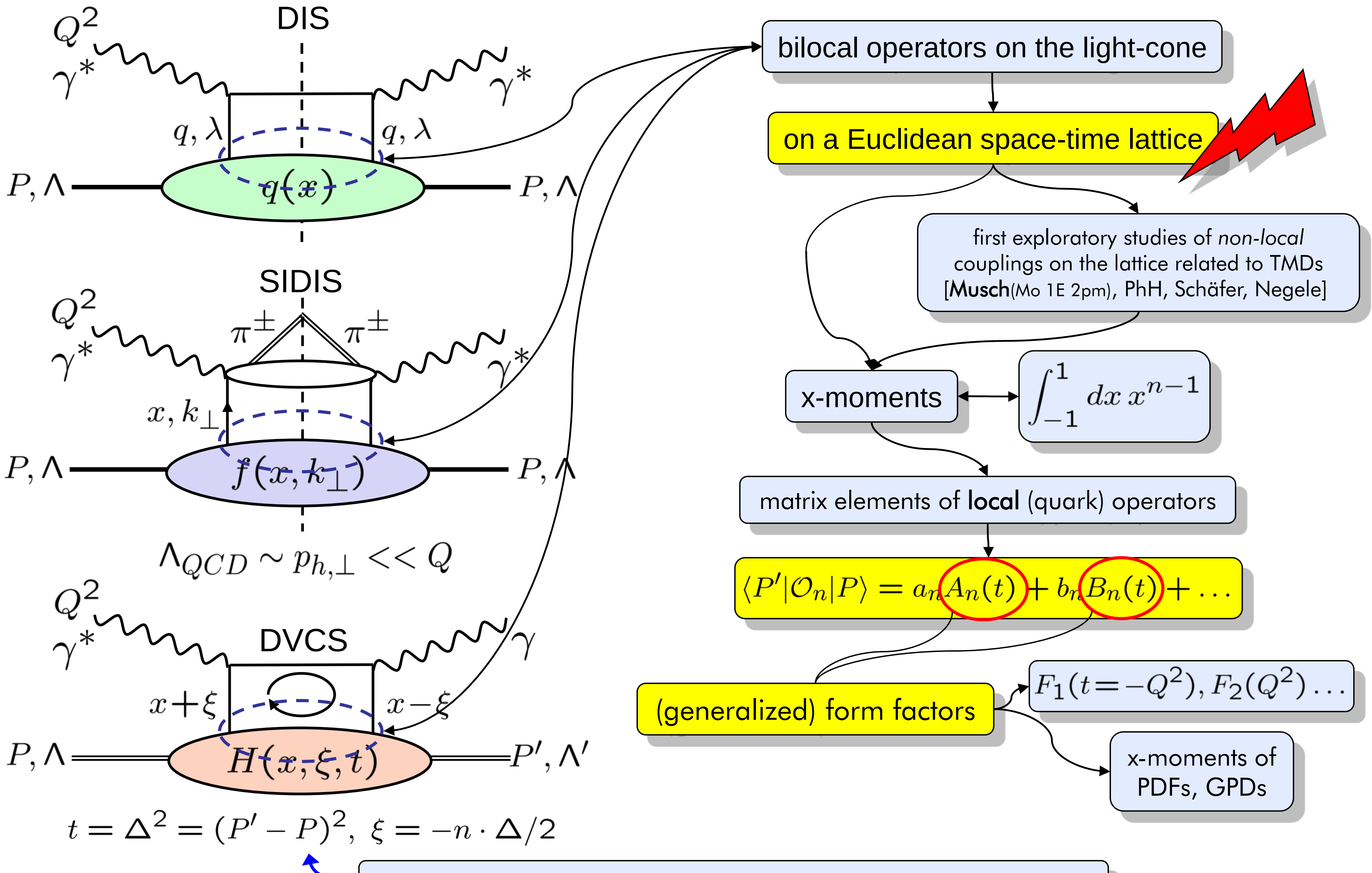
TMDs

$f_1(x, k_\perp^2), g_1, h_1, f_{1T}^\perp, g_{1T},$
$h_1^\perp, h_{1L}^\perp, h_{1T}^\perp, \ldots$

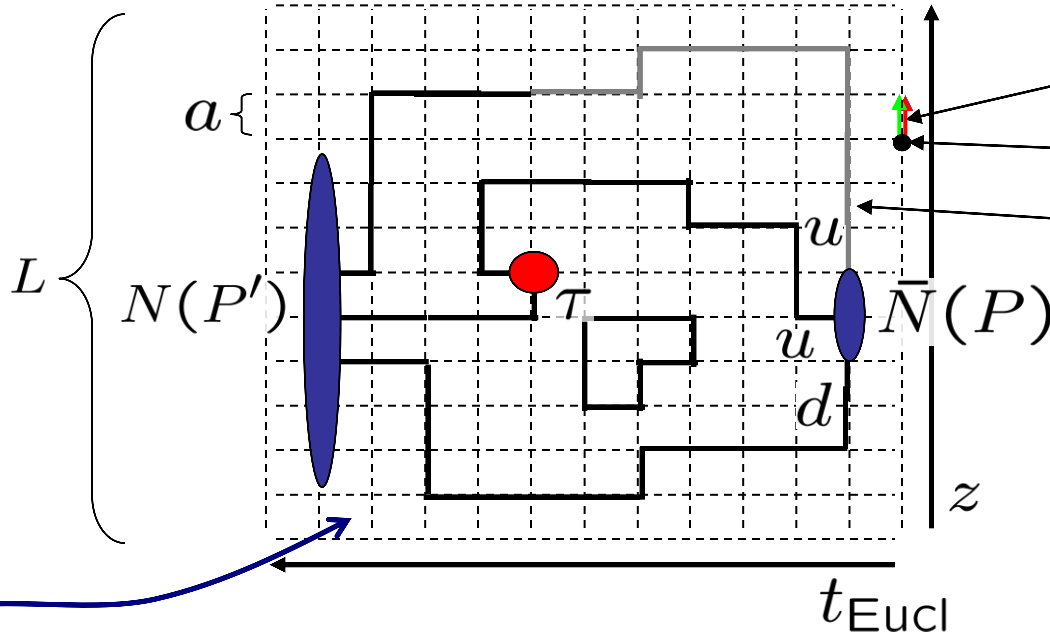
Distribution of momentum
in transverse momentum s

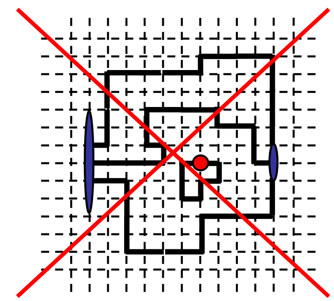
T-odd effects

QCD factorization and observables on the lattice



„Measurements“ on the lattice





irrel. for u-d

[Doi (Wed 6A 4:10pm)]

$$C_{3pt}(P', P, \tau) \leftrightarrow e^{-E'(T-\tau)-E\tau} \langle P', \Lambda' | \mathcal{O} | P, \Lambda \rangle \propto g_A, \Delta\Sigma, F_1(t), F_2(t), \langle x \rangle, A_{20}(t), \ldots$$

$$t = \Delta^2 (\hat{=} q^2) = (P' - P)^2$$

products of quark propagators ↔ compute the path-integral numerically using MC methods

Lattice QCD – general comments

systematic „ab initio“-approach, but

- statistical errors from MC integration
- discretization and finite volume errors/effects
- contaminations from excited states
- unphysical quark masses $m_\pi (\propto \sqrt{m_q}) \gtrsim 200$ MeV
- large minimal non-zero momenta $p_{\text{min}} = \frac{2\pi}{aL} \approx 300$ MeV

[☞**Aubin** (Wed 17:10pm)]
[☞**W.-Loud** (Wed 17:30pm)]
[☞**Edwards** (Fr 8:30am)]

Lattice QCD is different from model calculations

approximations can be continuously improved

mainly limited by computational and human resources

some „human resources" I am indebted to:

M. Altenbuchinger, B. Musch (→JLab), M. Gürtler (→Regensburg), W. Weise

(T39, TUM)

B. Bistrovic, J. Bratt, J.W. Negele,
A. Pochinsky, S. Syritsyn (MIT)
R.G. Edwards, B. Musch, D.G. Richards (JLab)
K. Orginos (W&M)
M. Engelhardt (New Mexico)
G. Fleming, M. Lin (Yale),
H.-W. Lin (INT),
H. Meyer (Mainz),
D.B. Renner (DESY Zeuthen),
M. Procura (TUM), W. Schroers

(LHPC)

D. Brömmel (Southampton),
M. Diehl (DESY),
M. Göckeler, M. Gürtler, Th. Hemmert,
A. Schäfer (Regensburg U.)
R. Horsley, J. Zanotti (Edinburgh U.)
Y. Nakamura (DESY Zeuthen)
P. Rakow (Liverpool U.)
D. Pleiter, G. Schierholz (DESY)
H. Stüben (ZIB)

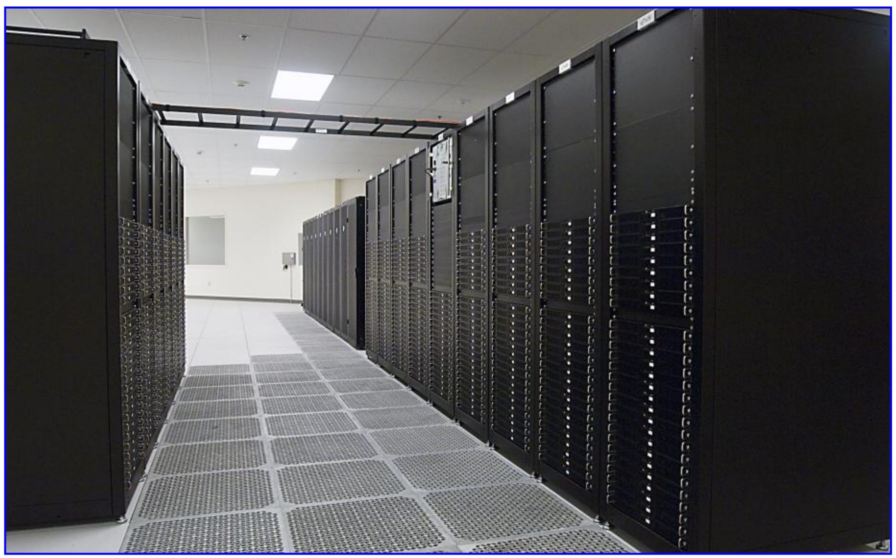
(QCDSF/UKQCD)

References: QCDSF PoS(LAT2006)120, 0710.1534, PRL 98 222001 (2007), PRL 2008 (0708.2249),
Brömmel et al EPJC 2007; LHPC PRD 77, 094502 (2008), 0810.1933; 1001.3620;
Diehl&Hägler EPJC hep-ph/0504175;
Musch et al. 0811.1536; Musch arXiv:0907.2381; PhH, Musch et al. EPL 2009 (arXiv:0908.1283)
PhH Phys.Rep. 2010 (0912.5483)

Machines

SGI Altix 4700 at LRZ Garching

APEmille at NIC/DESY Zeuthen



7n cluster at JLab

Blue Gene (MIT, EPCC,...)

sources: (http://www.lrz-muenchen.de/wir/einweihungsfeier/bildergalerie/_fotoindex.html) & Jlab, EPCC webpages

Overview

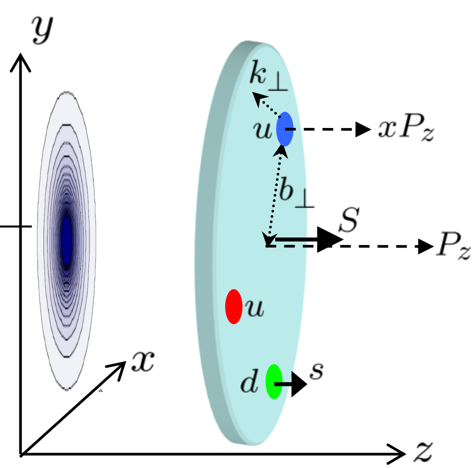
form factors – nucleon rms radii and (anomalous) magnetic moment — $F_1, F_2(Q^2) \rightarrow \langle r^2 \rangle_{1,2},\ \mu$

axial vector and tensor coupling constants — g_A, g_T

GPDs – generalized form factors and the form factors of the energy momentum tensor — $H, E(x, \xi, t) \rightarrow A(t),\ B(t),\ C(t)$

decomposition of the nucleon spin — $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$

correlations between momenta, positions and spins



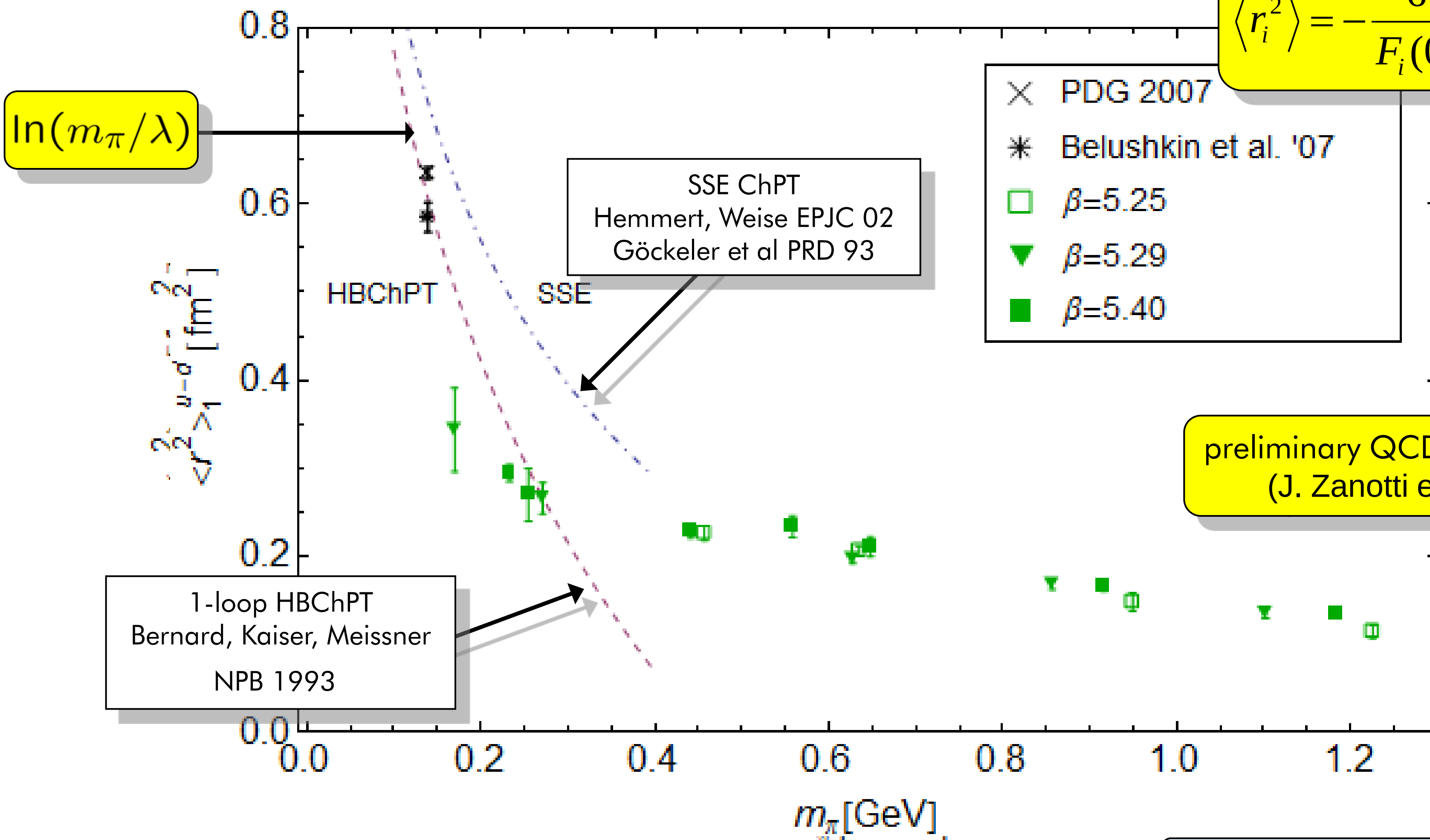
PhH Phys.Rep. 2010 (0912.5483)

Proton mean square radii – Dirac isovector radius

Dirac and Pauli FFs

$$\langle P'|\overline{q}\gamma_\mu q|P\rangle = \overline{U}(P')\left\{\gamma_\mu F_1(t) + i\frac{\sigma_{\mu\nu}\Delta^\nu}{2m_N}F_2(t)\right\}U(P)$$

$$\left\langle r_i^2\right\rangle = -\frac{6}{F_i(0)}\frac{d}{dt}F_i(t)\Big|_{t=0}$$

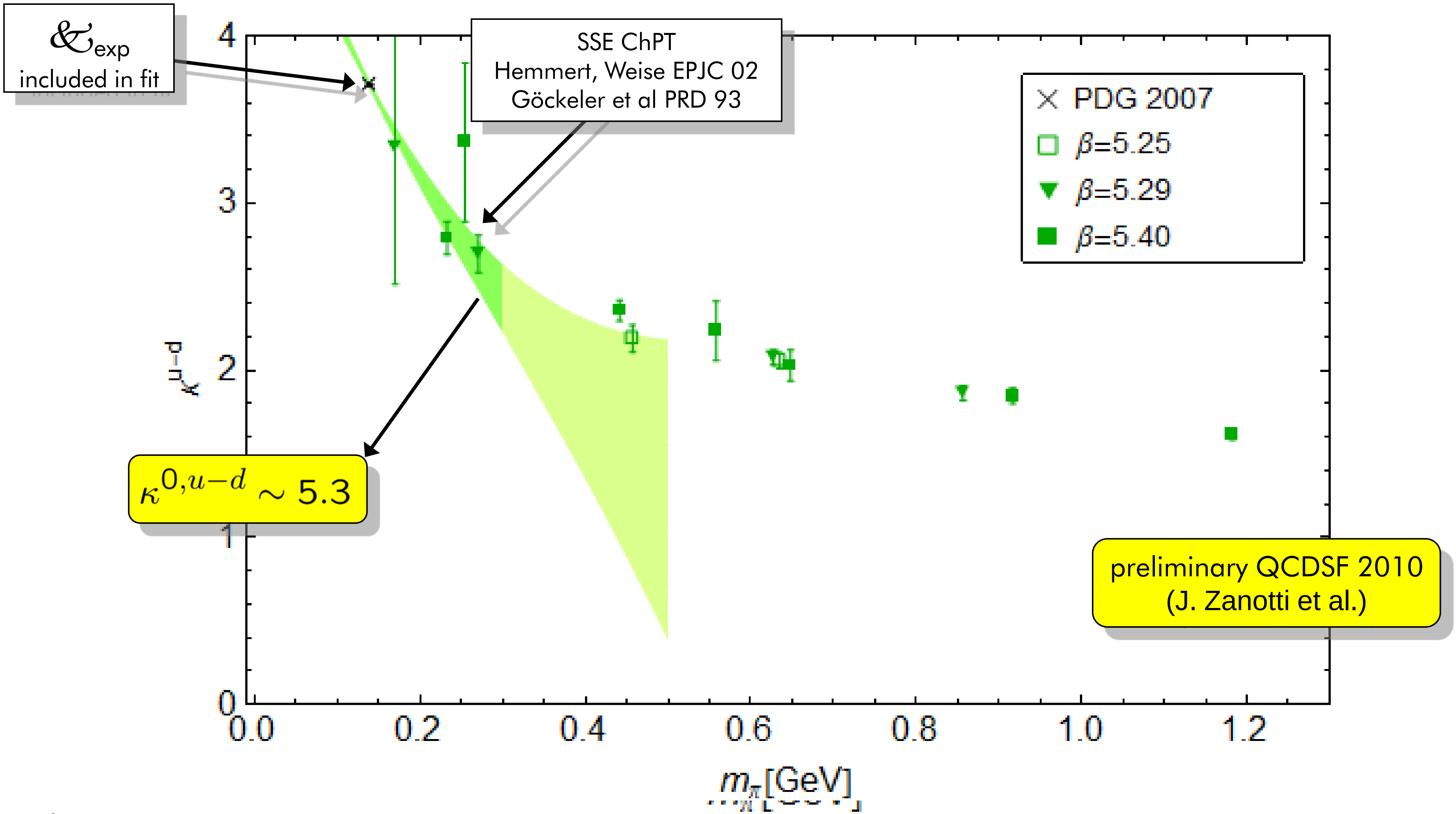


preliminary QCDSF 2010
(J. Zanotti et al.)

[☞Syritsyn(Mo 1E 3pm) for LHPC]

Nucleon isovector anomalous magnetic moment

(anomalous) magnetic moment → $\kappa = F_2(Q^2 \to 0) = \mu - F_1(0)$



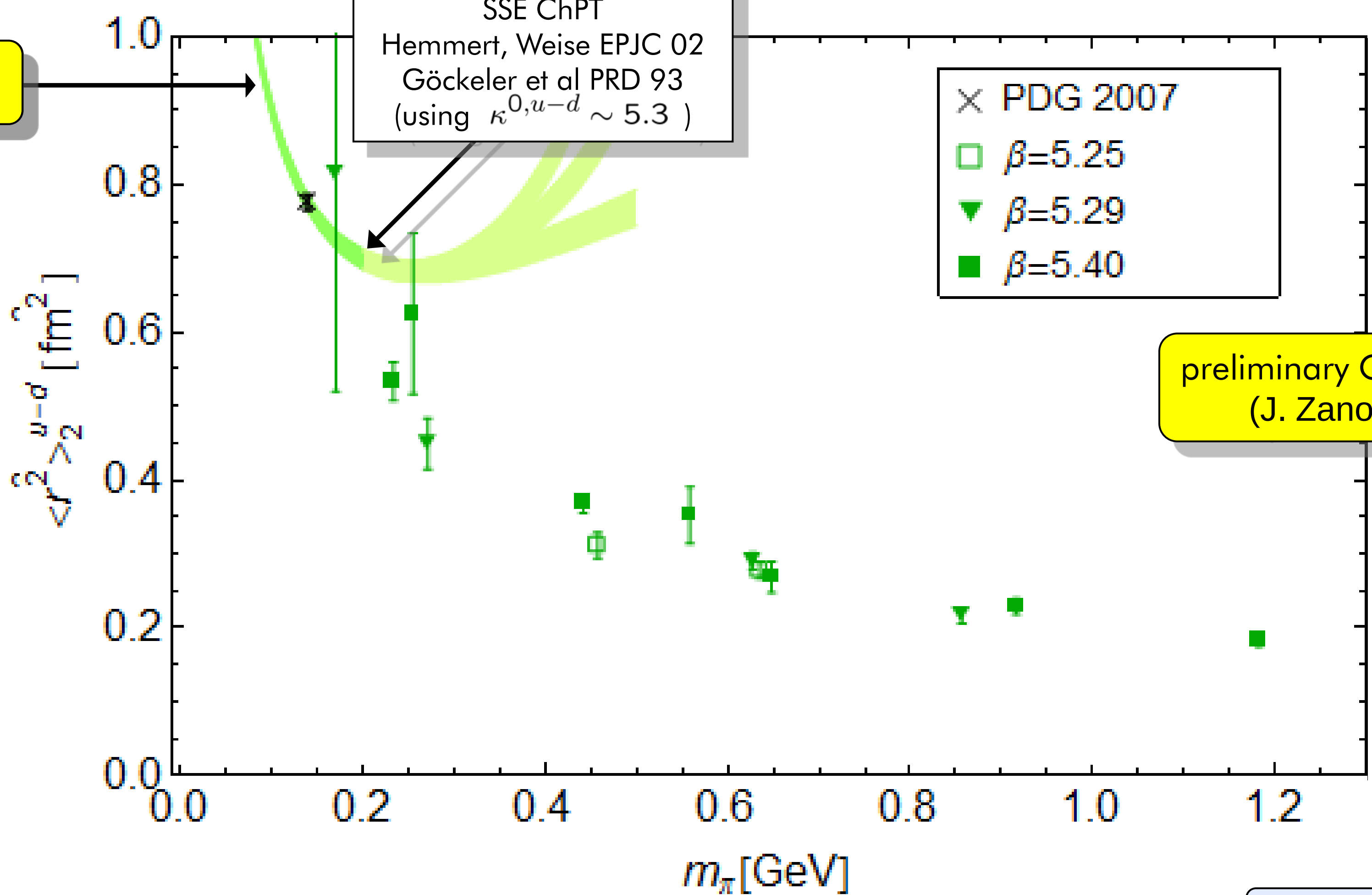
Proton mean square radii – Pauli isovector radius

Dirac and Pauli FFs

$$\langle P'|\overline{q}\gamma_\mu q|P\rangle = \overline{U}(P')\{\gamma_\mu F_1(t) + i\frac{\sigma_{\mu\nu}\Delta^\nu}{2m_N}F_2(t)\}U(P)$$

$1/m_\pi$

SSE ChPT
Hemmert, Weise EPJC 02
Göckeler et al PRD 93
(using $\kappa^{0,u-d} \sim 5.3$)

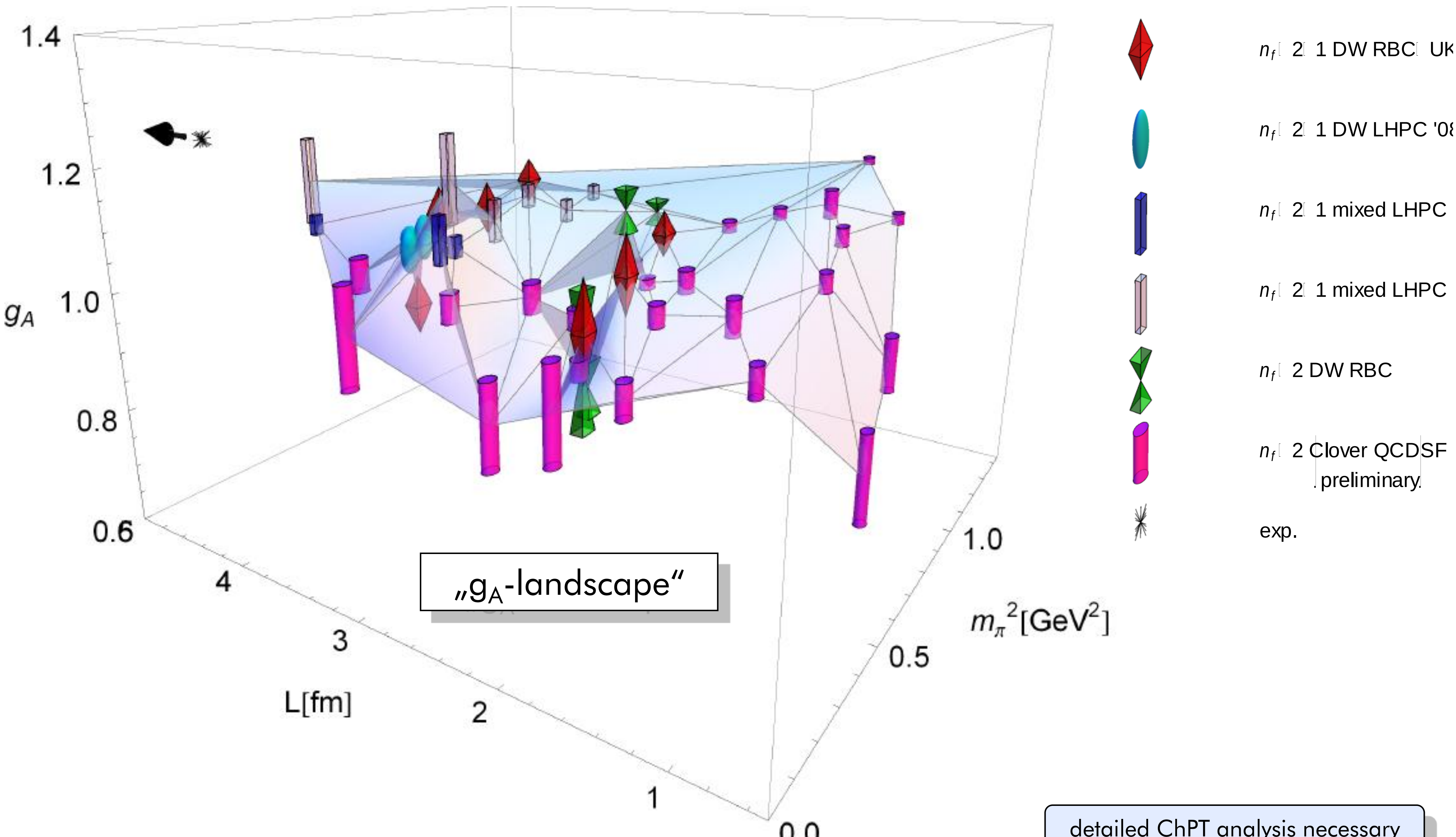


preliminary QCDSF 2010
(J. Zanotti et al.)

[☞Young (Fr 9am)]

Nucleon axial vector coupling constant

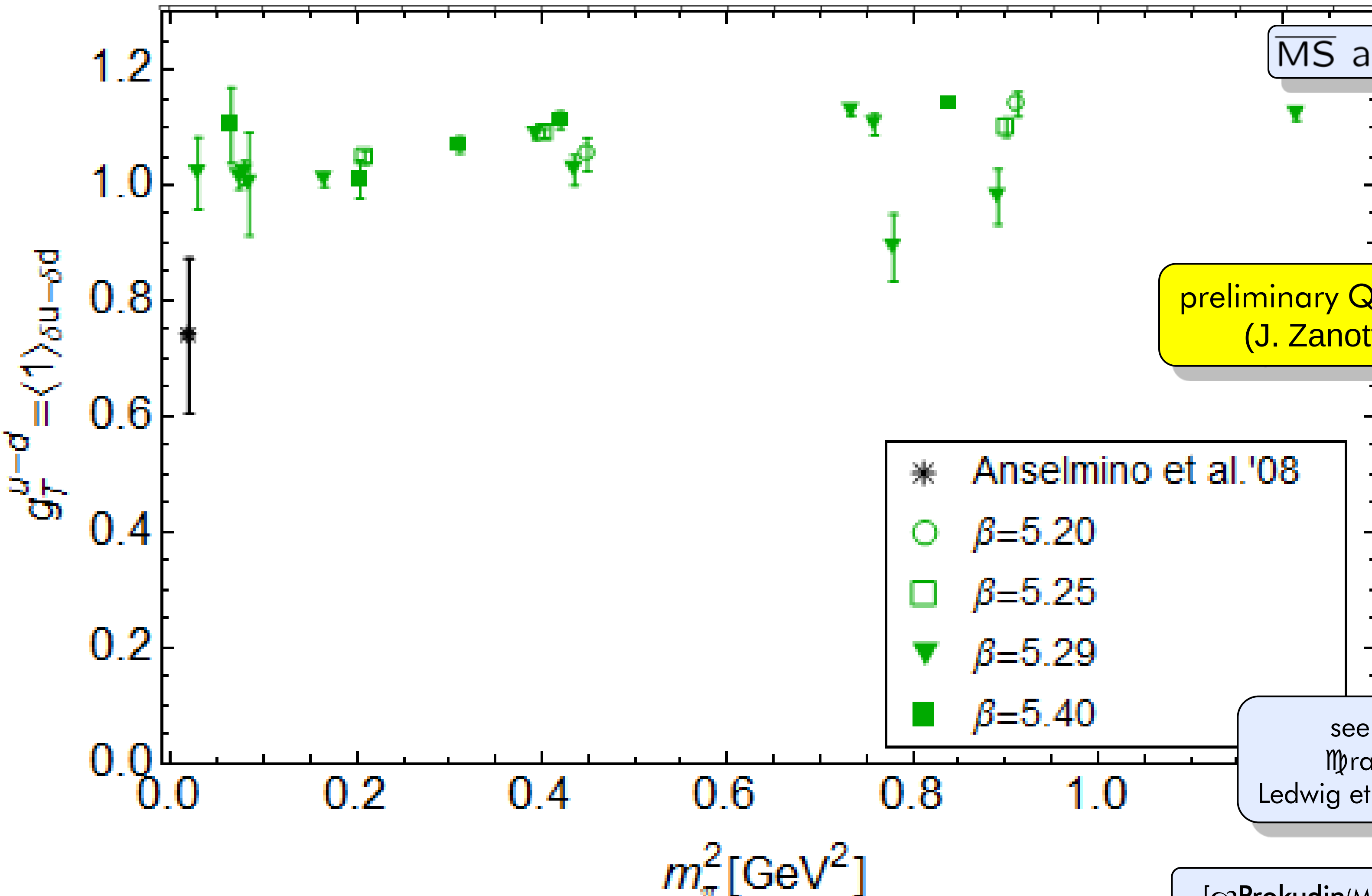
$$\langle P|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|P\rangle = g_A \bar{U}(P)\gamma_\mu\gamma_5 U(P)$$



detailed ChPT analysis necessary

Tensor charge

$$\langle P|\bar{q}i\sigma_{\mu\nu}q|P\rangle = g_T\bar{U}(P)i\sigma_{\mu\nu}U(P) \longleftrightarrow g_T = A_{T10}(0) = \int_{-1}^{+1} dx\,\delta q(x) = \langle 1\rangle_{\delta q} - \langle 1\rangle_{\delta\bar{q}}$$



preliminary QCDSF 2010
(J. Zanotti et al.)

see also recent
χiral QSM calc.
Ledwig et al. (1004.3612)

[☞**Prokudin**(Mo 1E 2:20pm)]

Higher (x^{n-1}-) moments of GPDs

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n-1} (-2\xi)^i A_{n,i}(t) + (-2\xi)^n C_{n,0}(t)\Big|_{n\ \text{even}}$$

$$\int_{-1}^{1} dx\, x^{n-1} E(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n-1} (-2\xi)^i B_{n,i}(t) - (-2\xi)^n C_{n,0}(t)\Big|_{n\ \text{even}}$$

GFFs

Ji&Lebed PRD 2000
Ph.H. PLB 2004

$t = \Delta^2 = (P' - P)^2,\ \xi = -n \cdot \Delta/2$

$n = 1$

$F_1(Q^2 = -t^2) = A_{10}(t)$
$F_2(Q^2 = -t^2) = B_{10}(t)$

$n = 2$

form factors of the EM-tensor

$\langle x^{n-1} \rangle_q = A^q_{n0}(0)$
$\langle x^{n-1} \rangle_{\Delta q} = \tilde{A}^q_{n0}(0)$

Decomposition of the nucleon spin

Ji's nucleon spin sum rule (X. Ji, PRL 1997)

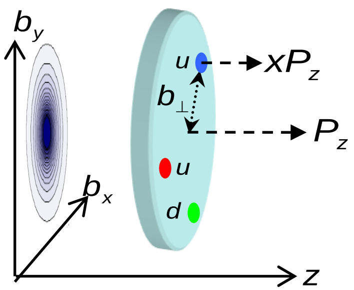
$$L_q \equiv J_q - \frac{1}{2}\Delta\Sigma_q$$
$$(L_g \equiv J_g - \Delta G)$$

$$\frac{1}{2} = S_z = \sum_{q,g} \frac{1}{2}\Big\{A_{20}(0) + B_{20}(0)\Big\}$$
$$= \sum_q J_q + J_g = \sum_q \frac{1}{2}\Delta\Sigma_q + \sum_q L_q + J_g$$

everything is:
-gauge-invariant
-scale & scheme dependent
-measurable

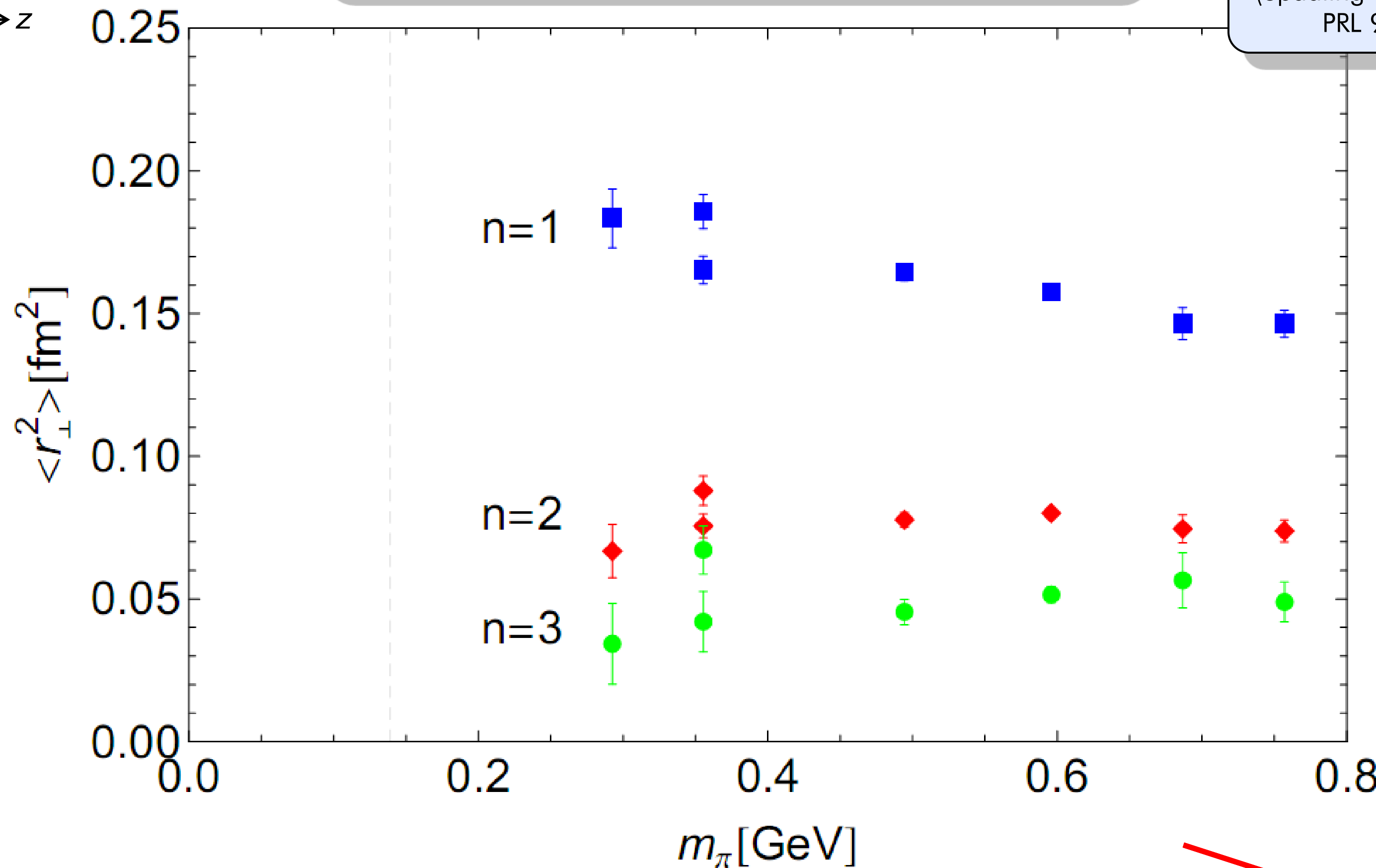
Generalized mean square radii of the nucleon

correlations in x and b_γ from n=1,2,3



$$\langle r_\perp^2 \rangle_n = \frac{\int d^2b_\perp b_\perp^2 \int dx x^{n-1} H(x,b_\perp)}{\int d^2b_\perp \int dx x^{n-1} H(x,b_\perp)}$$

LHPC n_f=2+1 mixed; arXiv:1001.3620 (updating PRD 2008, 0810.1933 and PRL 93, hep-lat/0312014)



$\overline{\langle x \rangle} \approx 0.2$

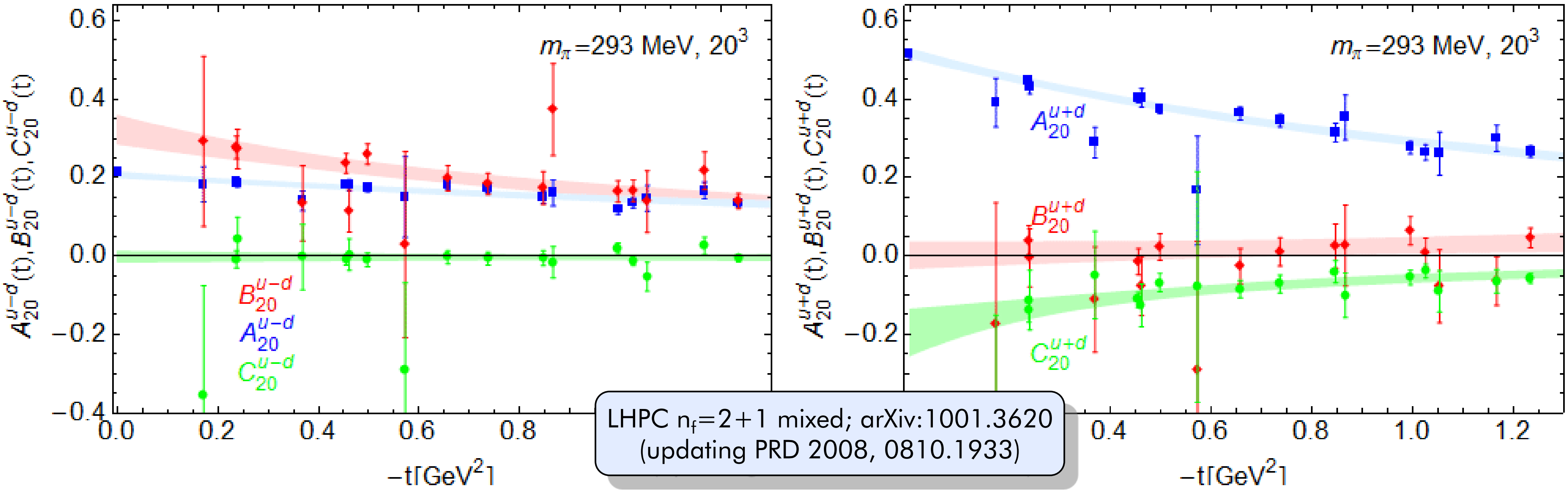
$\overline{\langle x \rangle} \approx 0.25$

$\overline{\langle x \rangle} \approx 0.4$

strong correlations in x and b_γ ↔ no factorization of GPDs in x and t ↔ ~~$H(x,0,t) = f(x)h(t)$~~

n=2 - A, B, C - Form factors of the energy momentum tensor

$$\langle P'|\overline{q}\gamma^{\{\mu}D^{\nu\}}q|P\rangle = \langle P'|T_q^{\mu\nu}|P\rangle = \overline{U}(P')\Big\{\gamma^{\{\mu}\overline{P}^{\nu\}}A_{20}(t) - \frac{i\Delta_\rho\sigma^{\rho\{\mu}\overline{P}^{\nu\}}}{2m_N}B_{20}(t) + \frac{\Delta^\mu\Delta^\nu}{m_N}C_{20}(t)\Big\}U(P)$$



LHPC n_f=2+1 mixed; arXiv:1001.3620
(updating PRD 2008, 0810.1933)

$$B_{20}^{u-d} > A_{20}^{u-d}$$
$$C_{20}^{u-d} \approx 0$$

$$A_{20}^{u+d} > B_{20}^{u+d}$$
$$C_{20}^{u+d} < 0$$

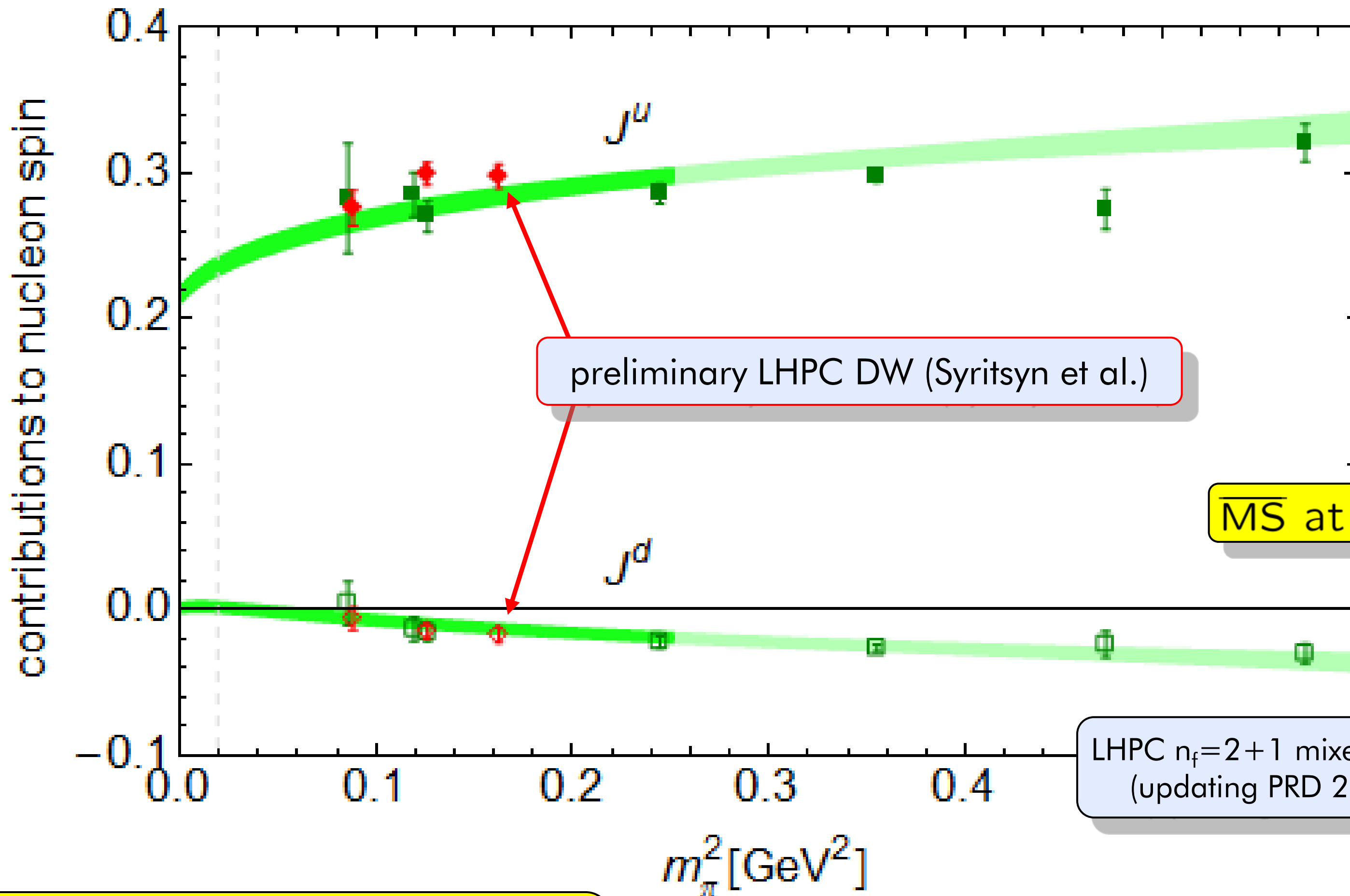
seems to be compatible with large N_c limit
– see e.g. Goeke, Polyakov, Vanderhaeghen PiPaNP 2001

disconnected contributions are not included↔ only u-d is „exact"

$\overline{MS}$ at 4 GeV^2

Quark angular momentum

$J_q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$ — from covariant BChPT extrapolations — Dorati, Gail, Hemmert NPA 2008



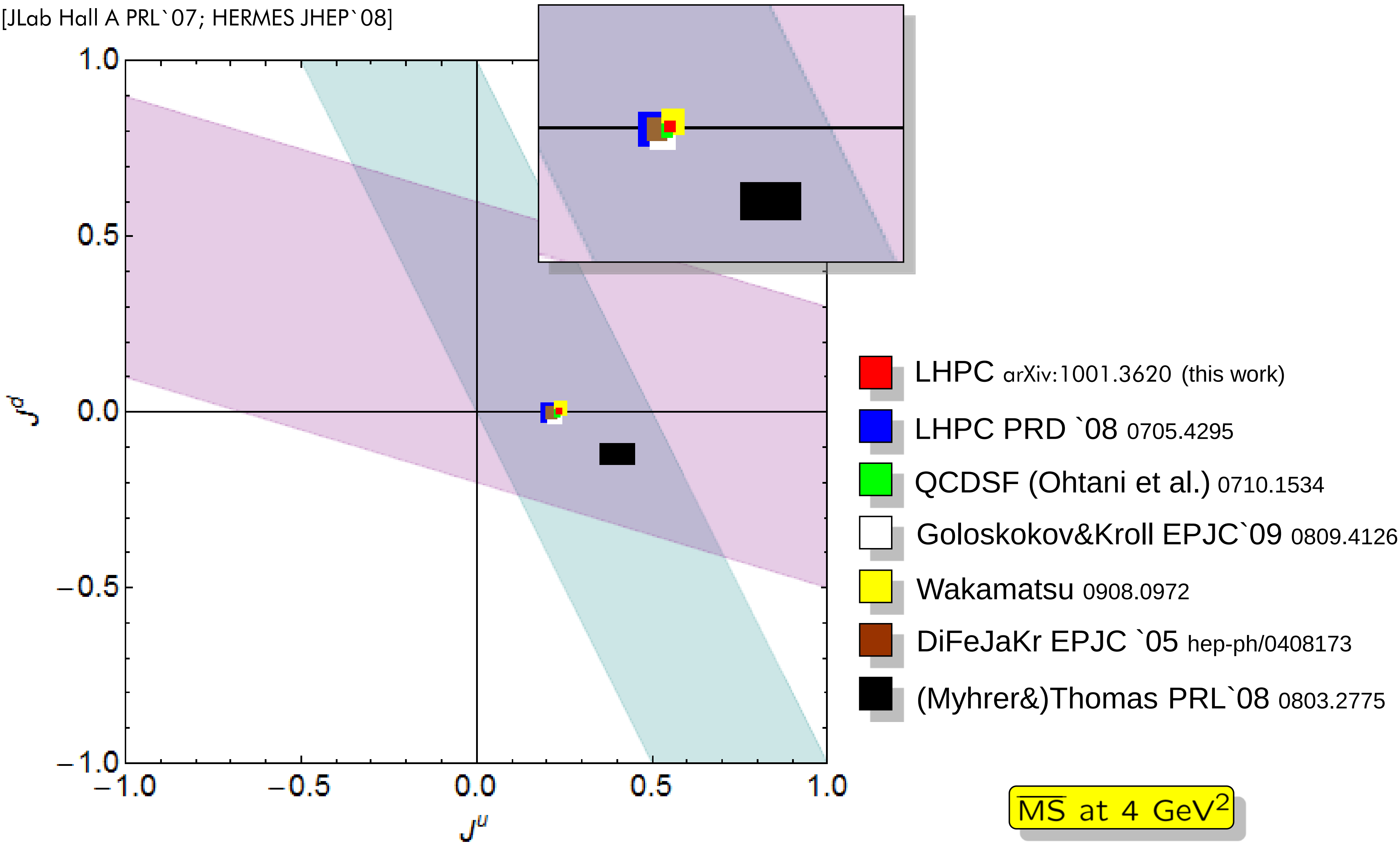
LHPC n_f=2+1 mixed; arXiv:1001.3620
(updating PRD 2008, 0810.1933)

$J^u = 0.236(6) \approx 47\%$ of 1/2
$J^d = 0.0018(37) \approx 1\%$ of 1/2

→ $J^{u+d} \approx 0.238 \pm 0.008 \approx 48\%$ of 1/2

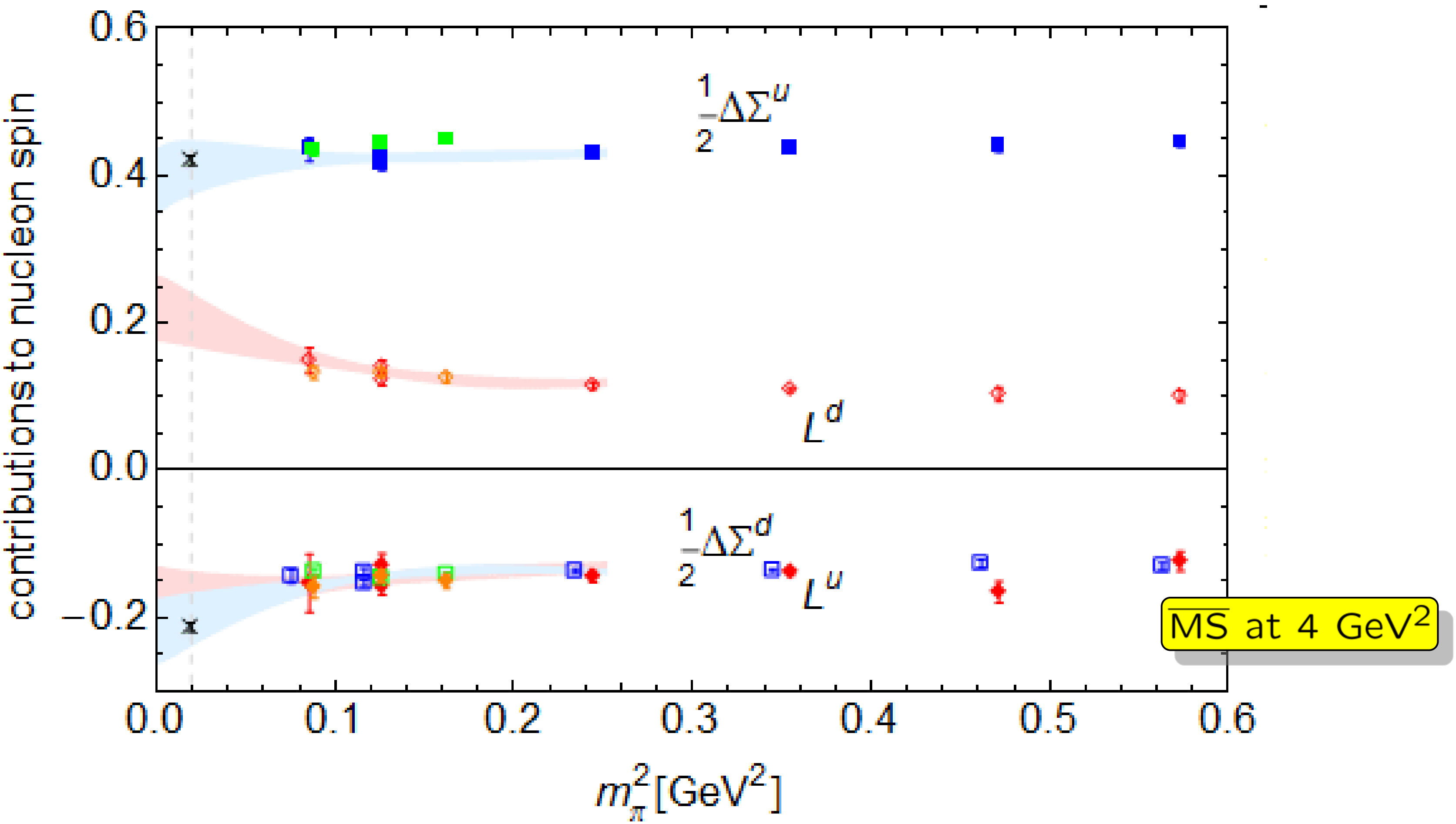
J_u, J_d template figure

[JLab Hall A PRL`07; HERMES JHEP`08]



$\overline{MS}$ at 4 GeV^2

Quark spin and orbital angular momentum



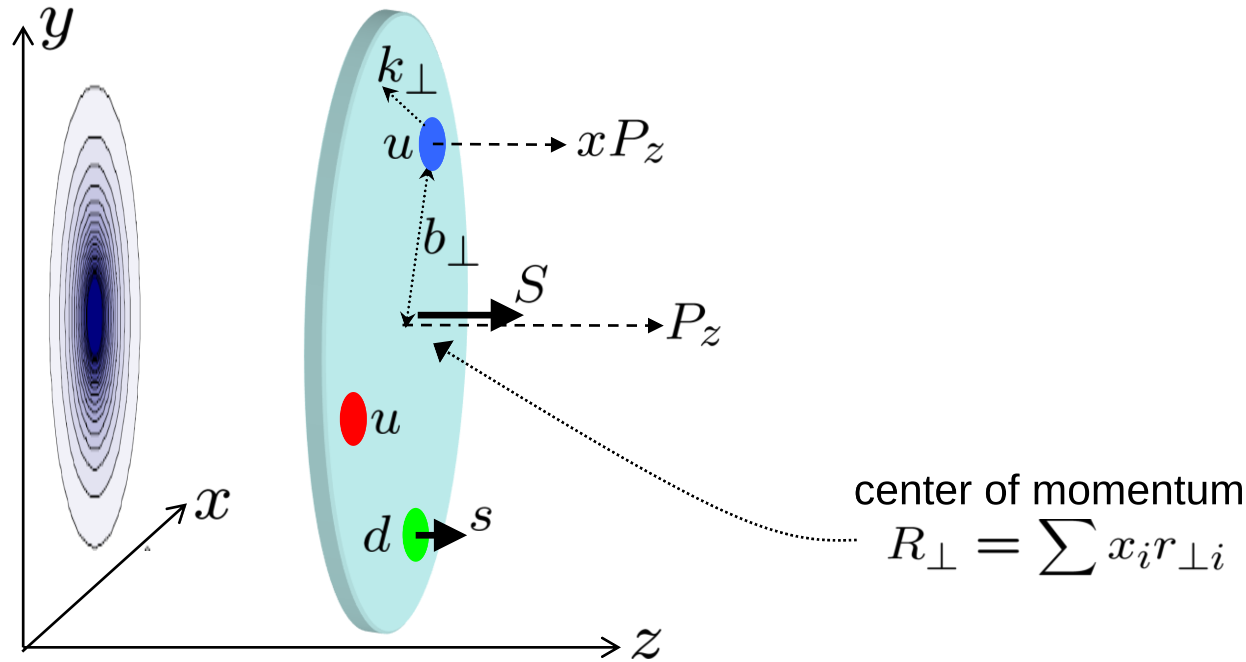
$J^u \approx 0.236 \pm 0.006 \,\hat{=}\, 48\%$ of $1/2$
$J^d \approx 0.002 \pm 0.004$

pioneering lattice calculations by Gadiyak, Ji and Jung in 2001

$L^d \approx -L^u \approx 0.185 \pm 0.06 \approx 36\%$ of $1/2$
$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of $1/2$

$\kappa^{u+d} = 3\kappa^{p+n} = -0.36$

Correlations between momenta, positions, spins



generalized parton distributions (GPDs)

$H(x, b_\perp) \overset{\text{FT}}{\leftrightarrow} H(x, \xi = 0, t)$

$b_\perp \leftrightarrow -\Delta_\perp \widehat{=} |t|^{\frac{1}{2}}$

Burkardt PRD 2000

~~Fourier-transformation~~

transverse momentum dependent PDFs

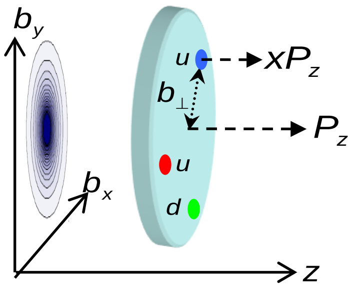
$f(x, k_\perp) \overset{\text{FT}}{\leftrightarrow} A(l \cdot P, l^2)$

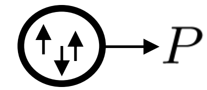
$k_\perp \leftrightarrow l_\perp$
$x \leftrightarrow l \cdot P$

$\langle P | \overline{q}(l) \Gamma U q(0) | P \rangle$

Transverse spin densities in the proton

in impact parameter space





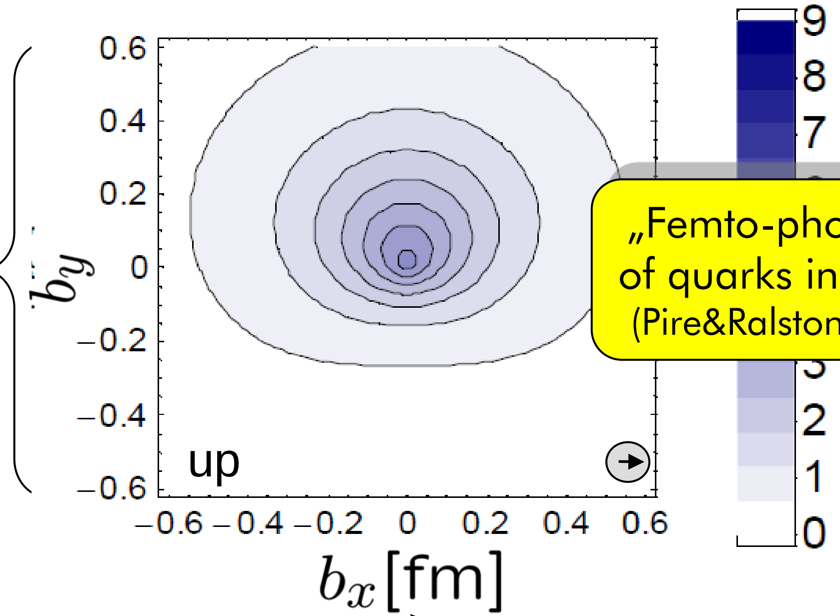
$$\rho_T(x, b_\perp; s_\perp) = \frac{1}{2}\left\{H(x, b_\perp^2) - \epsilon_{ij} s_\perp^i b_\perp^i \frac{1}{m_N} E_T{}'(x, b_\perp^2)\right\}$$

Diehl / PhH EPJC 2005

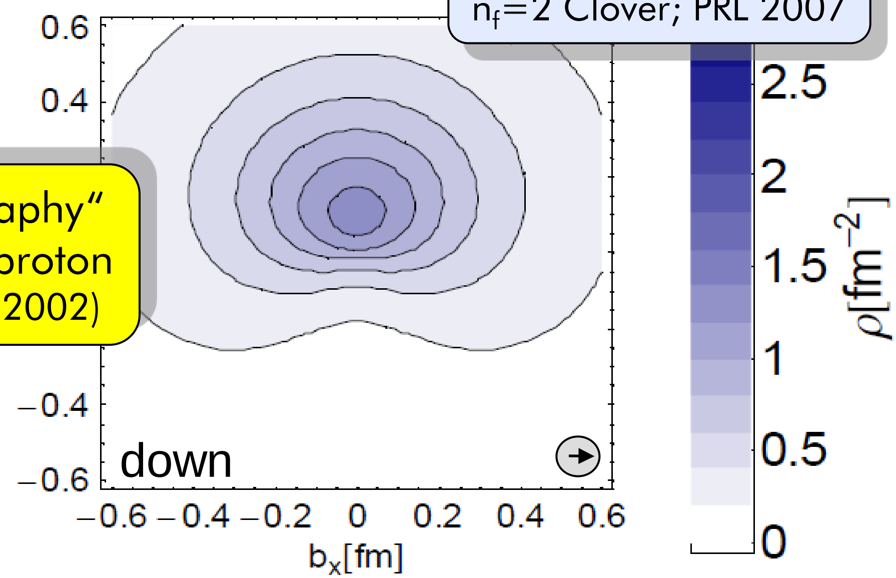
lattice calculations of quark *spin-flip couplings*

quark transverse spin in (+x)-direction

QCDSF (PhH et al.)
n_f=2 Clover; PRL 2007



„Femto-photography"
of quarks in the proton
(Pire&Ralston PRD 2002)



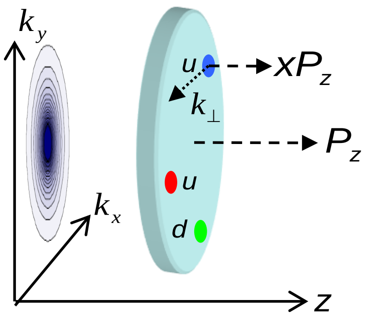
$\overline{\text{MS}}$ at 4 GeV^2

strongly deformed **perpendicular** to transverse spin

Intrinsic transverse momentum densities of the nucleon

first exploratory lattice study of transverse momentum distributions of quarks in the nucleon using non-local operators

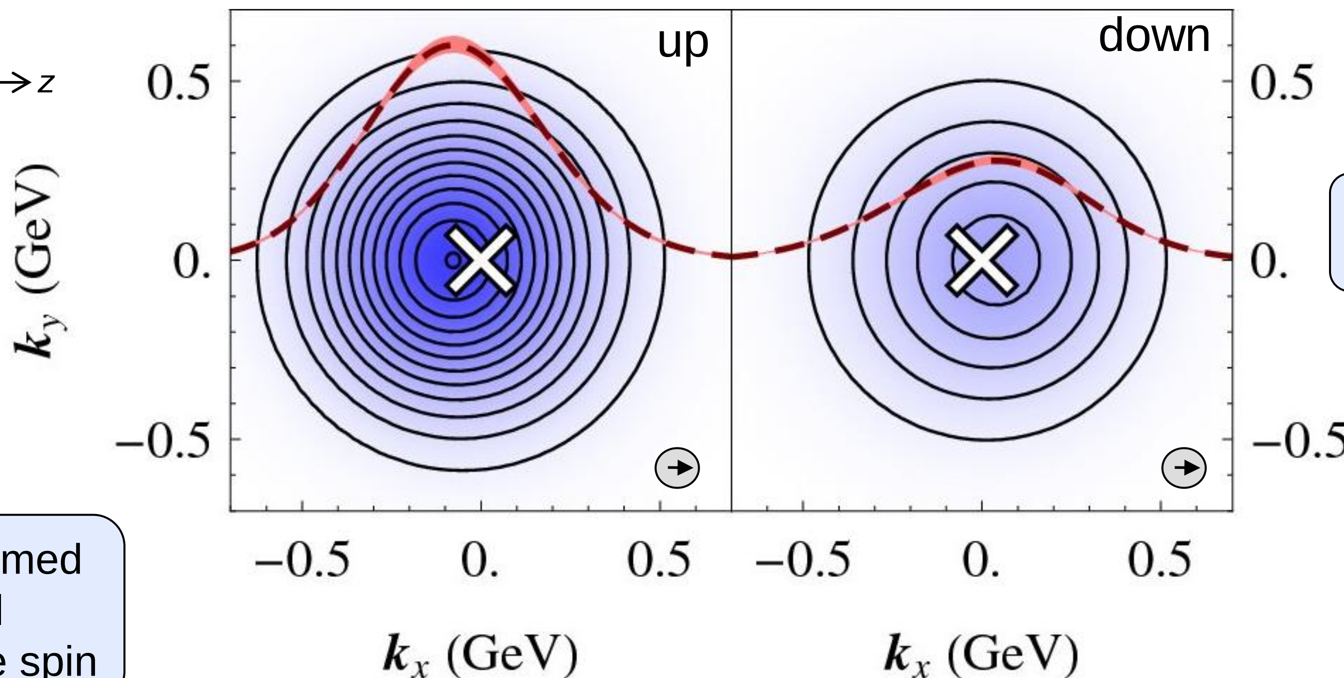
not identical to TMDs in exp.



$$\rho(x, k_\perp; \Lambda, s_\perp) = \frac{1}{2}\left(f_1 + \Lambda \frac{k_\perp \cdot s_\perp}{m_N} h_{1L}^\perp\right)$$

Diehl, PhH EPJC 44 (2005)

Boglione, Mulders PRD 60 (1999)



PhH, **B. Musch** et al. EPL; arXiv:0908.1283

[**Musch**(Mo 1E 2pm)

visibly deformed **parallel** to transverse spin

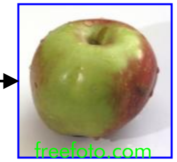
~~$h_{1L}^\perp(x, k_\perp) \leftrightarrow$ GPDs~~

genuine effect of intrinsic transverse momentum of quarks

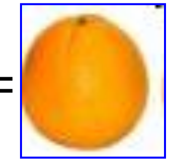
Summary

remarkable progress in lattice QCD calculations of hadron structure

Dirac and Pauli radii → tension with (H)BChPT? ... or ↔



$\stackrel{?}{=}$



large values for tensor charge → tension with TMD-phenomenology

$$\frac{1}{2} \stackrel{*}{\approx} 0.238(8)_{[J^{u+d}]} + J_g = 0.208(10)_{[\Delta\Sigma^{u+d}/2]} + 0.030(12)_{[L^{u+d}]} + J_g$$

$$= 0.411(36)_{[\Delta\Sigma^{u}/2]} - 0.175(36)_{[L^{u}]} - 0.203(35)_{[\Delta\Sigma^{d}/2]} + 0.205(35)_{[L^{d}]} + J_g$$

$\overline{\text{MS}}$ at 4 GeV^2

strong correlations between coordinate, momentum & spin DOFs → evidence for substantial genuine intrinsic k_T, ...

Challenges

systematic uncertainties in „benchmark observables" (radii, magnetic moment, g_A,...)

disconnected diagrams; gluon operators; operator mixing

*[non-singlet, connected only; additional uncertainties due to chiral extrapolations, renormalization]

under investigation (Syritsyn et al.)

Backup

Decomposition of the proton spin

$\overline{\text{MS}}$ at 4 GeV^2

	J^{u-d}	J^{u+d}	J^u	J^d
BChPT	0.234(6)	0.238(8)	0.236(6)	0.0018(37)
HBChPT		0.264(6)		
HBChPT + Δ		0.226(22)		
mixed ChPT				
experiment				

	$g_A = \Delta\Sigma^{u-d}$	$\Delta\Sigma^{u+d}/2$	$\Delta\Sigma^u/2$	$\Delta\Sigma^d/2$	L^{u-d}	L^{u+d}	L^u	L^d
BChPT								
HBChPT		0.208(10)				0.056(11)		
HBChPT + Δ	1.21(17)							
mixed ChPT			0.411(36)	−0.203(35)	−0.379(71)	0.030(12)	−0.175(36)	0.205(35)
experiment	1.2670(35)	0.208(9)	0.421(6)	−0.214(6)				

$$\frac{1}{2} \overset{*}{\approx} 0.238(8)_{[J^{u+d}]} + J_g = 0.208(10)_{[\Delta\Sigma^{u+d}/2]} + 0.030(12)_{[L^{u+d}]} + J_g$$

$$= 0.411(36)_{[\Delta\Sigma^u/2]} - 0.175(36)_{[L^u]} - 0.203(35)_{[\Delta\Sigma^d/2]} + 0.205(35)_{[L^d]} + J_g$$

$\overline{\text{MS}}$ at 4 GeV^2

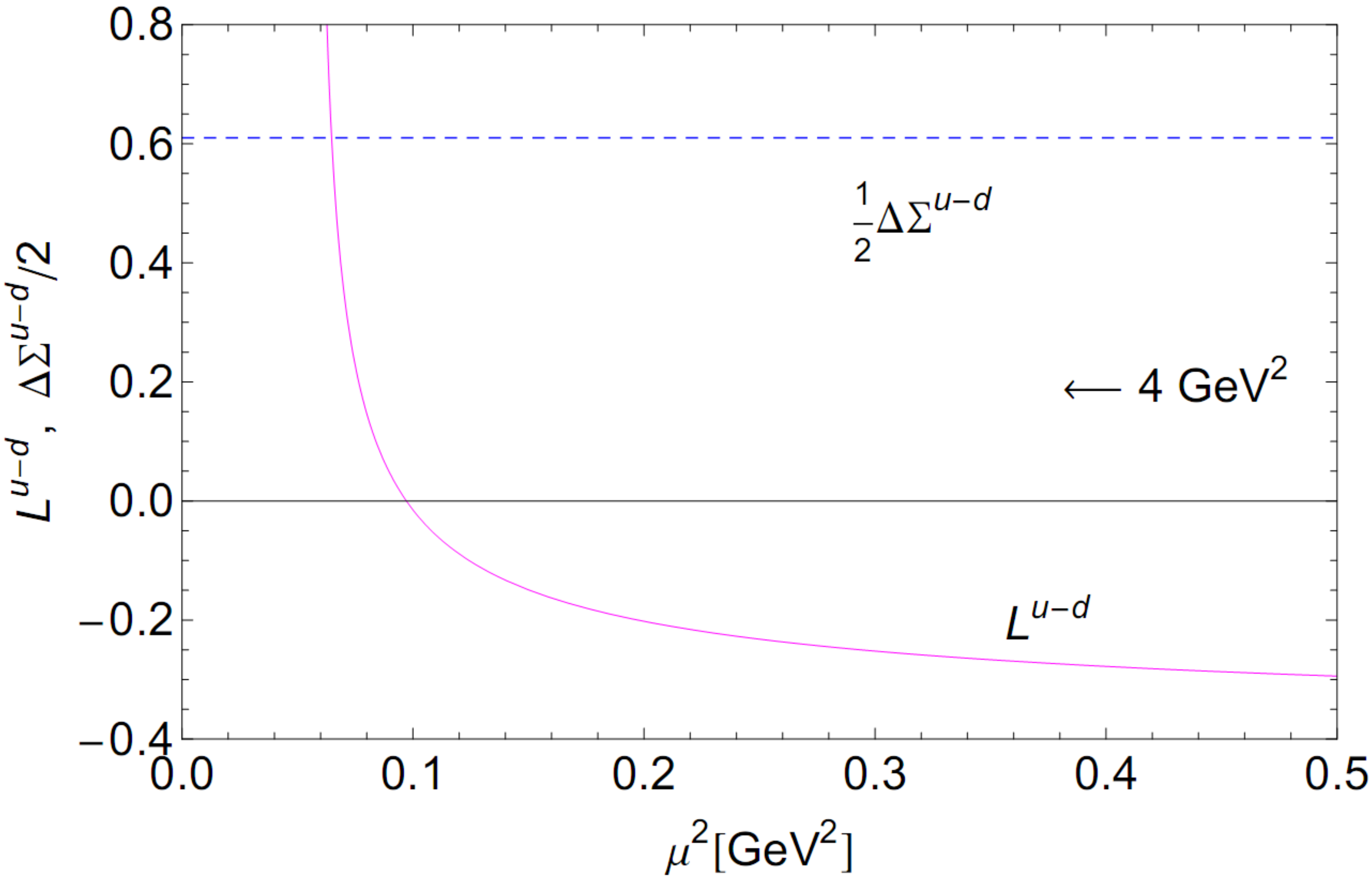
*[non-singlet, connected only; additional uncertainties due to chiral extrapolations, renormalization]

under investigation (Syritsyn et al.)

Lattice QCD vs relativistic quark models – QCD evolution

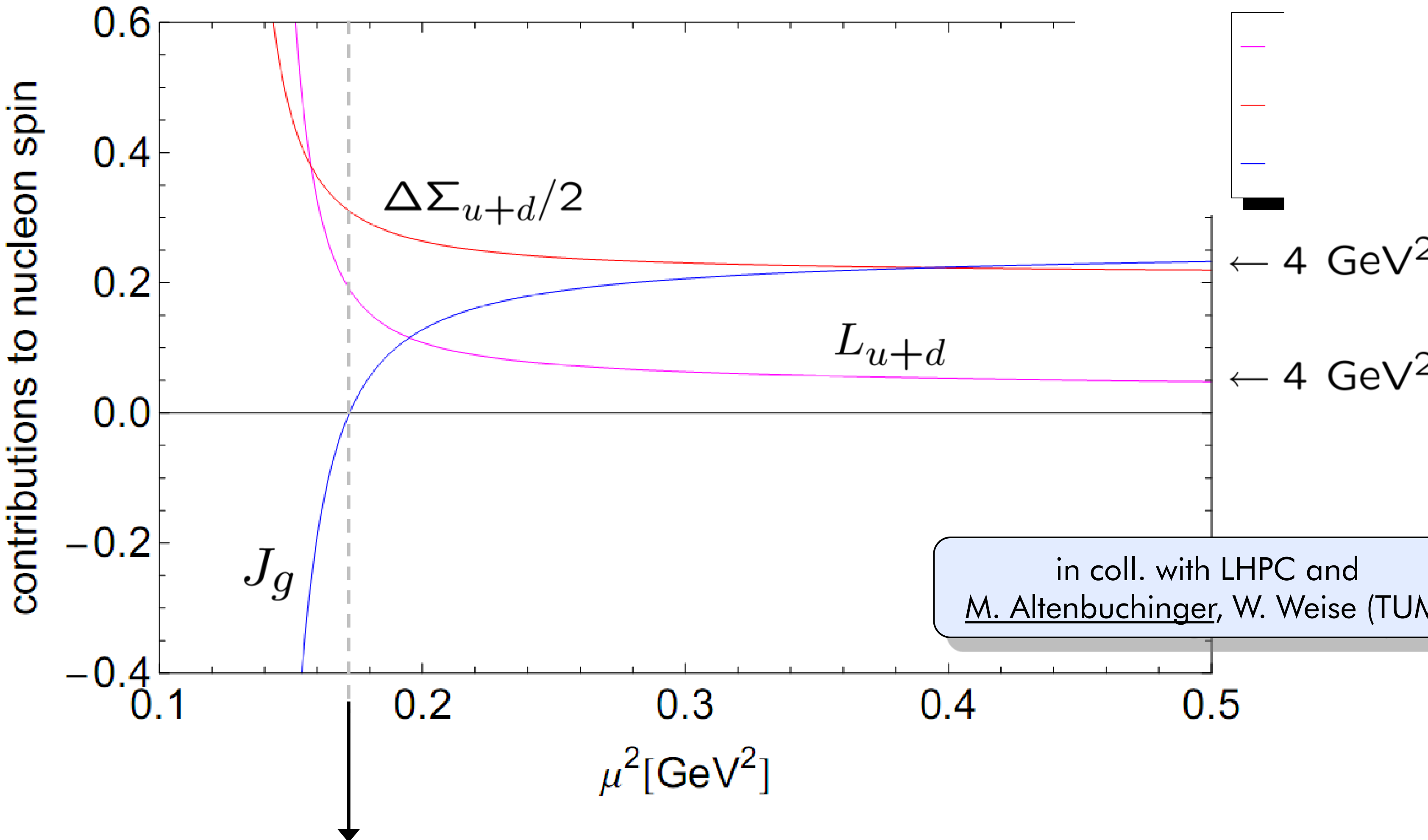
(Wakamatsu 2005; Thomas, PRL 2008)

$$L^{u-d}(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{\beta_0}\left(\frac{-16}{9}\right)} \left\{L^{u-d}(t_0) + \frac{1}{2}\Delta\Sigma^{u-d}\right\} - \frac{1}{2}\Delta\Sigma^{u-d}$$



Lattice QCD vs relativistic quark models – QCD evolution

(Wakamatsu 2005; Thomas, PRL 2008)



in coll. with LHPC and
M. Altenbuchinger, W. Weise (TUM)

$L^{u+d} \approx 0.19 \approx 38\%$ of $1/2$
$\Delta\Sigma^{u+d}/2 \approx 0.31 \approx 62\%$ of $1/2$

lattice + evolution

$L^{u+d} \approx 0.18 \approx 36\%$ of $1/2$
$\Delta\Sigma^{u+d}/2 \approx 0.32 \approx 64\%$ of $1/2$

relativistic quark model

Lattice simulation details

- mixed action approach: DW fermions on a Asqtad staggered sea for $N_f=2+1$; including HYP-smearing
- $L_s=16$, m_{res}⊙$0.1 m_q$
- lattice spacing a ~ 0.124 fm
- volumes of ~2.5 and ~3.5 fm^3
- two sink momenta P'=(0,0,0), (-1,0,0)

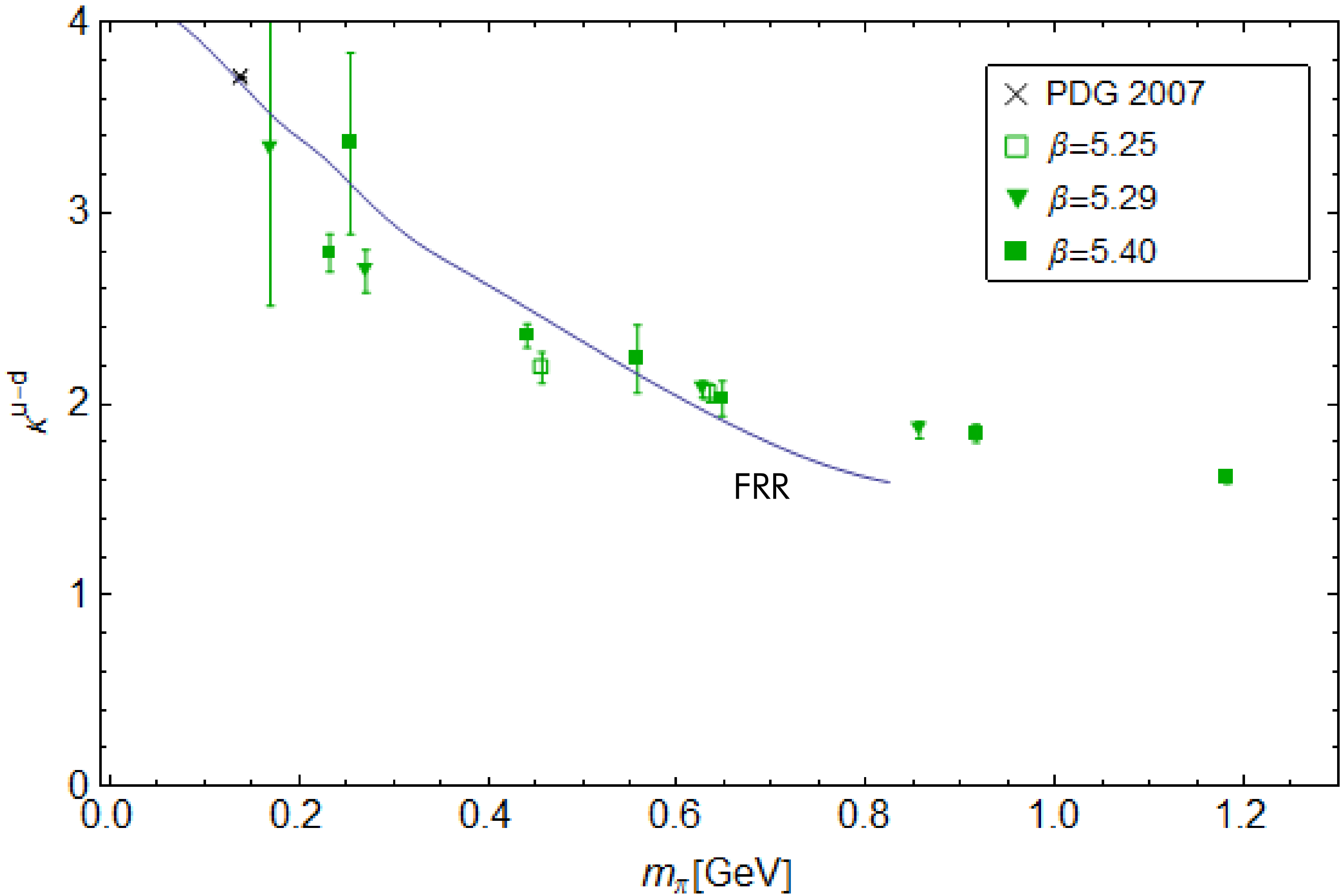
operator renormalization: $Z_{\mathcal{O}} = \frac{Z_{\mathcal{O}}^{pert}}{Z_A^{pert}} Z_A^{nonpert}$

$\overline{MS}$ at 4 GeV^2

Light m_{sea}^{asqtad}	Volume Ω	$(am)_\pi$	$(af)_\pi$	$(am)_N$	m_π [MeV]	f_π [MeV]	m_N [MeV]
0.007	$20^3 \times 64$	0.1842(7)	0.0657(3)	0.696(7)	292.99(111)	104.49(45)	1107.1(111)
0.010	$28^3 \times 64$	0.2238(5)	0.0681(2)	0.726(5)	355.98(80)	108.31(34)	1154.8(80)
0.010	$20^3 \times 64$	0.2238(5)	0.0681(2)	0.726(5)	355.98(80)	108.31(34)	1154.8(80)
0.020	$20^3 \times 64$	0.3113(4)	0.0725(1)	0.810(5)	495.15(64)	115.40(23)	1288.4(80)
0.030	$20^3 \times 64$	0.3752(5)	0.0761(2)	0.878(5)	596.79(80)	121.02(34)	1396.5(80)
0.040	$20^3 \times 32$	0.4325(12)	0.0800(5)	0.941(6)	687.94(191)	127.21(78)	1496.8(95)
0.050	$20^3 \times 32$	0.4767(10)	0.0822(4)	0.991(5)	758.24(159)	130.70(67)	1576.3(80)

of „measurements" increased by factor 8 compared to PRD 77 094502 (2008)

ongoing efforts within LHPC based on DW fermions (RBC/UKQCD) and improved Wilson fermions (BMW)



PDG 2007
β=5.25
β=5.29
β=5.40
FRR
κ^{u-d}
m_π[GeV]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
1
2
3
4

Gluon contributions to the proton spin

$L_g \equiv J_g - \Delta G$

☹ Burkardt et al. RPP 73:016201,2010

$\frac{1}{2} \overset{*}{\approx} 0.238(8)_{[J^{u+d}]} + \Delta G + L_g$

$\overline{\text{MS}}$ at 4 GeV^2

DSSV PRL 101:072001,2008;
PRD 80:034030,2009

$\Delta G \approx \int_{0.001}^{1} \Delta g(x) = -0.035 \pm 0.2_{(?)}$

$\overline{\text{MS}}$ at 4 GeV^2

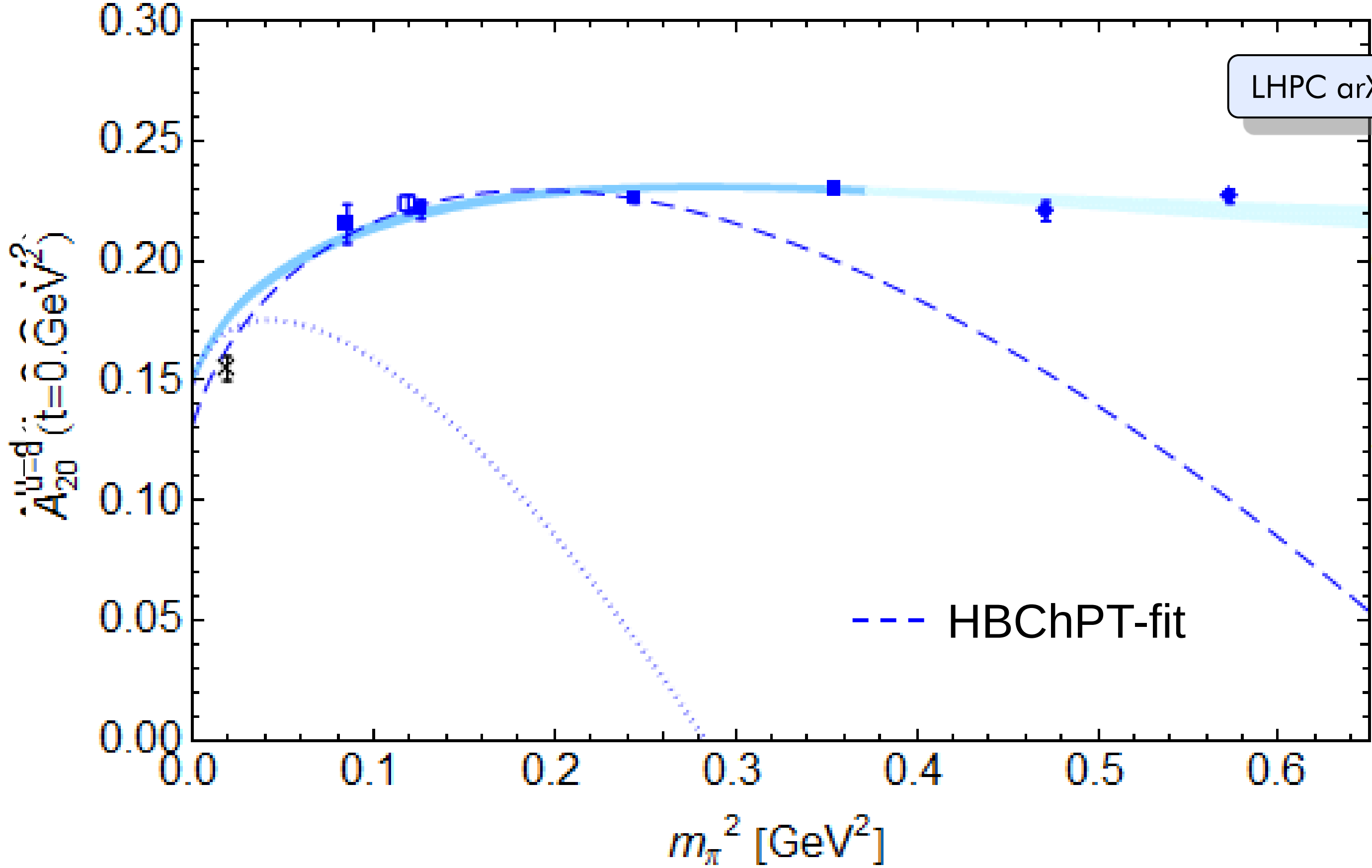
$L_g \approx 0.3 \pm 0.3$ (?)

* [non-singlet, connected only; additional uncertainties due to chiral extrapolations, renormalization]

Form factors of the energy momentum tensor

isovector quark momentum fraction

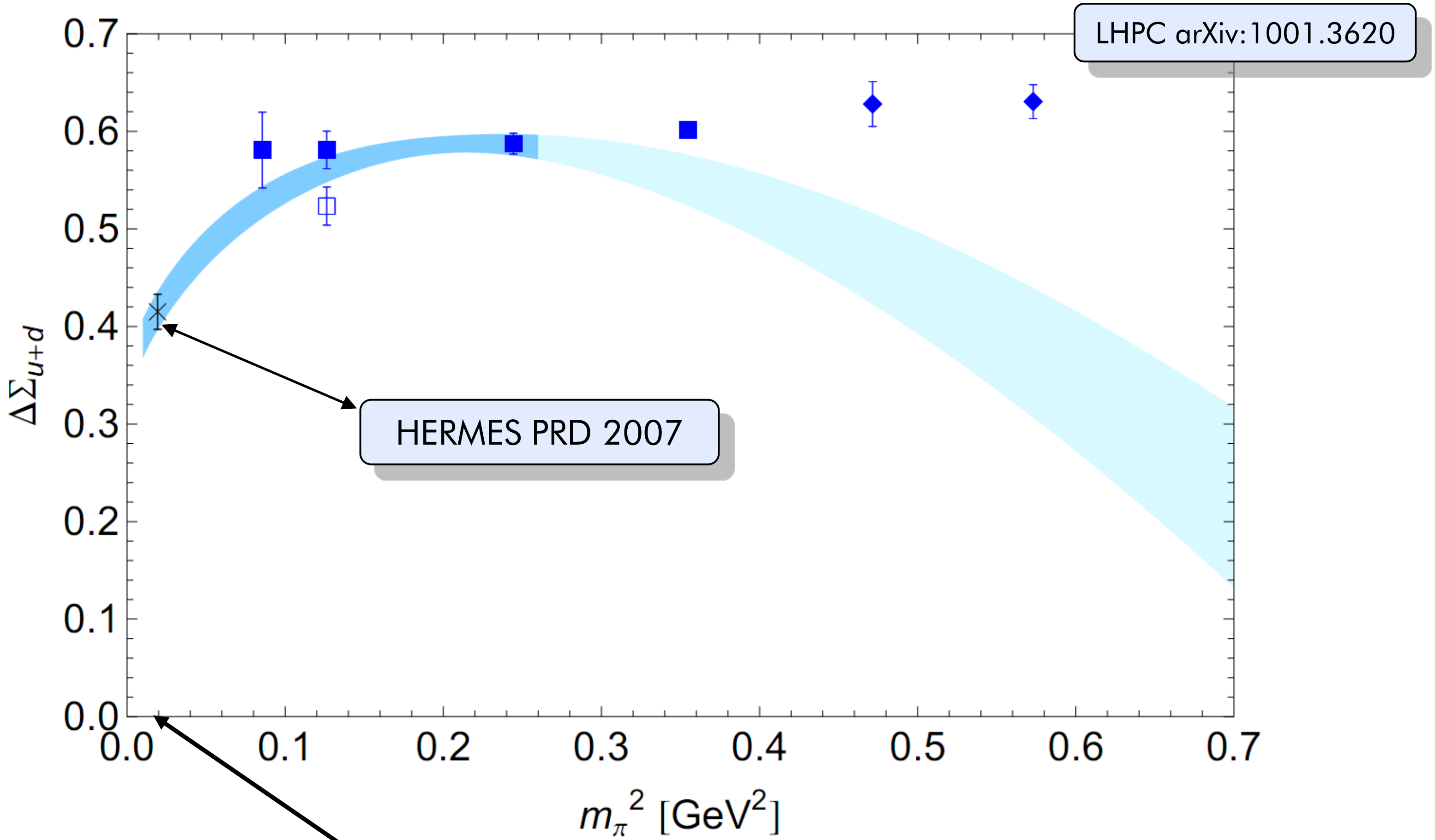
compared to LHPC PRD 77 094502 (2008)



Isosinglet quark spin fraction

(required for L=J-ΔΣ/2)

employing HBChPT by Diehl, Manashov, Schäfer EJPA 2006; Ando, Chen, Kao PRD 2006

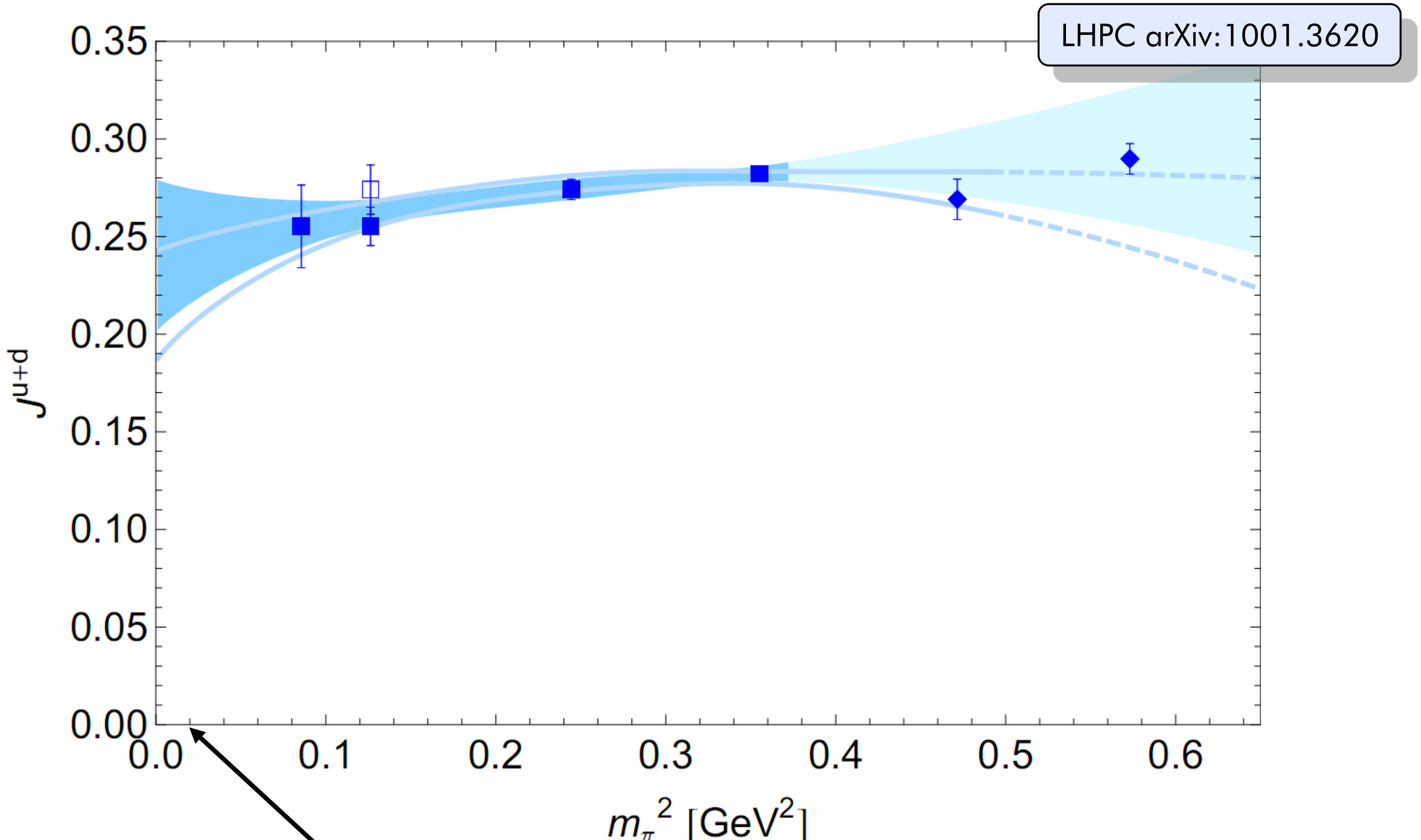


$\Delta\Sigma^{u+d}/2 = 0.208(10)$

$\overline{MS}$ at 4 GeV^2

Quark angular momentum

emplyoing HBChPT+👎 results [Chen Ji PRL 2002]



$J^{u+d} \approx 0.245(30) \approx 50\%$ of $1/2$

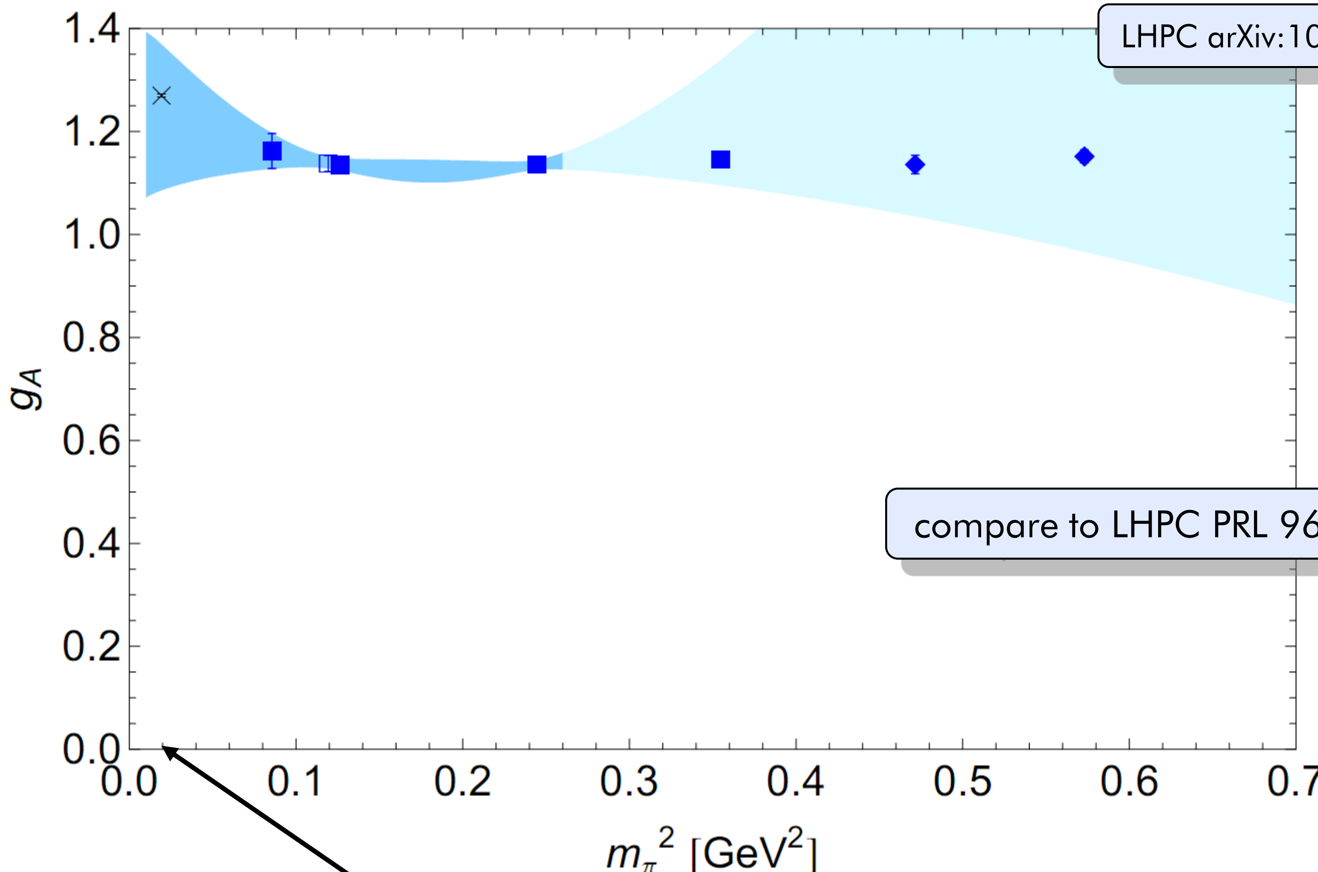
$\overline{\text{MS}}$ at 4 GeV^2

Isovector axial vector coupling constant

(required for $L=J-\Delta\Sigma/2$)

emplyoing SSE (HBChPT+Δ) results [Procura, Hemmert, Musch, Weise PRD 2007, QCDSF PRD 2006]

LHPC arXiv:1001.3620



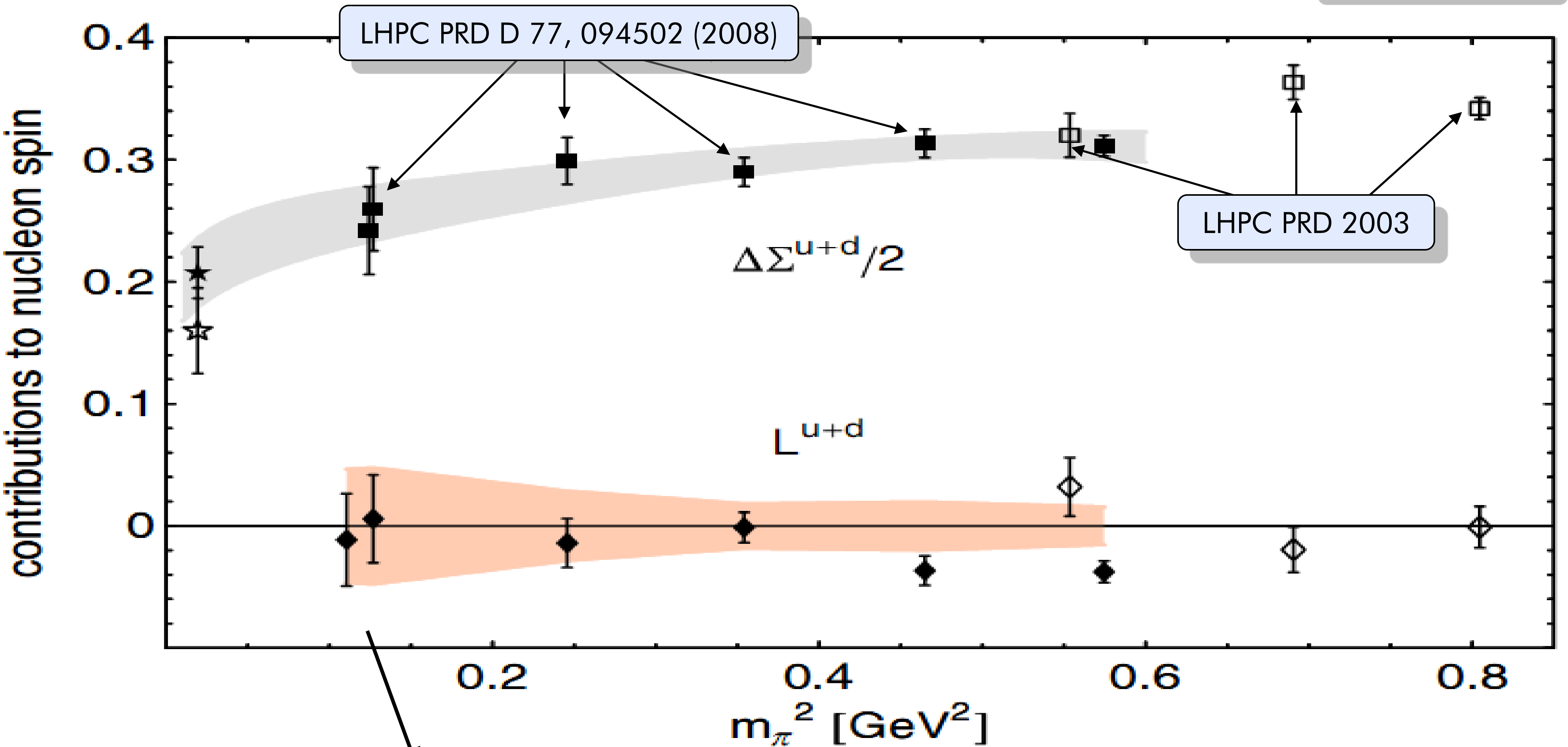
compare to LHPC PRL 96 502001 (2006)

$$g_A^3 = \Delta\Sigma^{u-d} = 1.21(17)$$

Quark spin and OAM

$J_q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$ — $L_q \equiv J_q - \Delta\Sigma_q/2$

$\overline{\text{MS}}$ at 4 GeV^2

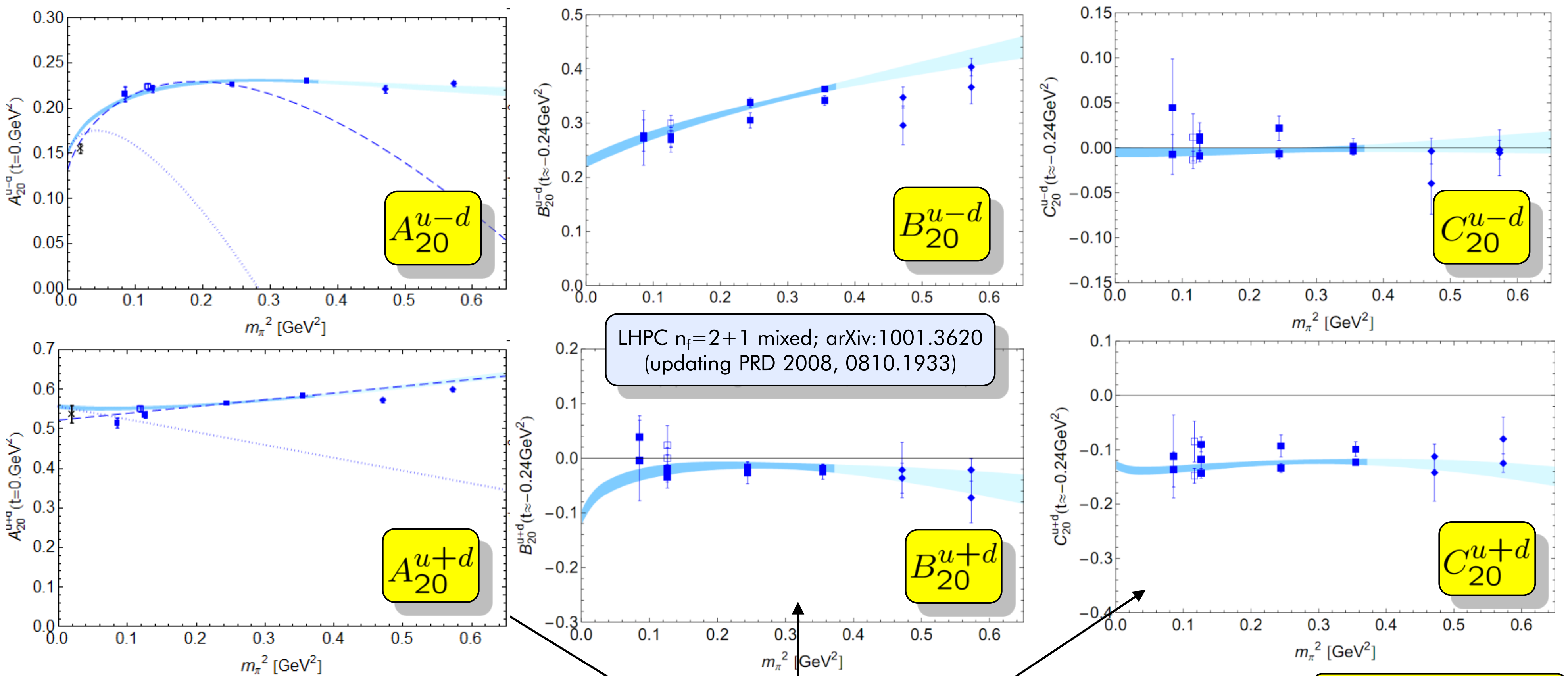


$\Delta\Sigma^{u+d}/2 \approx 0.21 \pm 0.006 \approx 42\%$ of $1/2$

$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of $1/2$

→ $J^{u+d} \approx 0.238 \pm 0.008 \approx 48\%$ of $1/2$

Global, simultaneous chiral extrapolation of A, B, C



LHPC $n_f=2+1$ mixed; arXiv:1001.3620 (updating PRD 2008, 0810.1933)

only quark line connected contributions

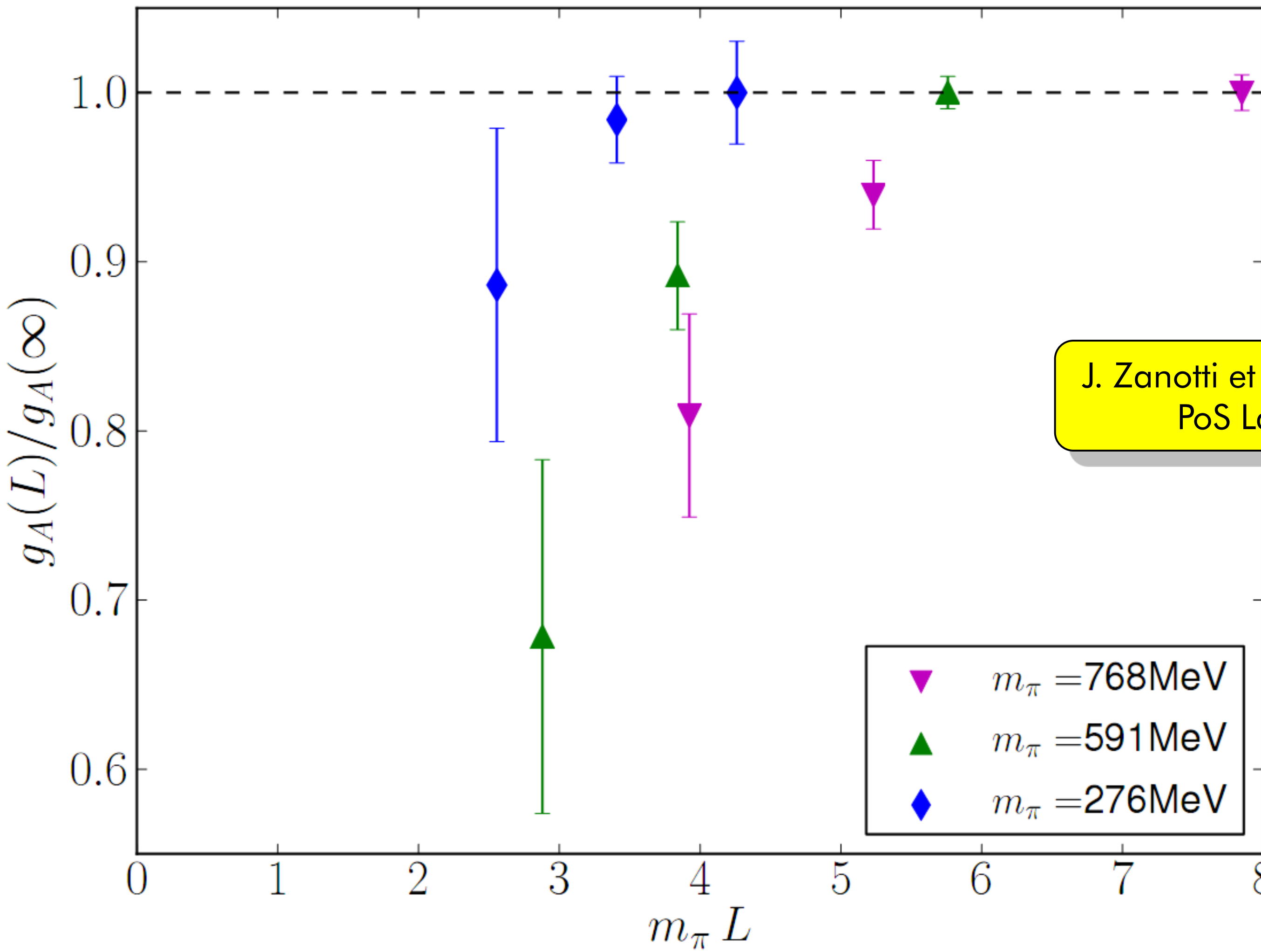
$\overline{\text{MS}}$ at 4 GeV^2

chiral extrapolation based on covariant BChPT by Dorati, Gail, Hemmert NPA 2008

$A_{20}(t, m_\pi), B_{20}(t, m_\pi), C_{20}(t, m_\pi)$ → with common parameter → $A_{20}^0 = A_{20}(t=0, m_\pi=0) = \langle x \rangle^0$

Nucleon axial vector coupling constant

$$\langle P|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|P\rangle = g_A \bar{U}(P)\gamma_\mu\gamma_5 U(P)$$



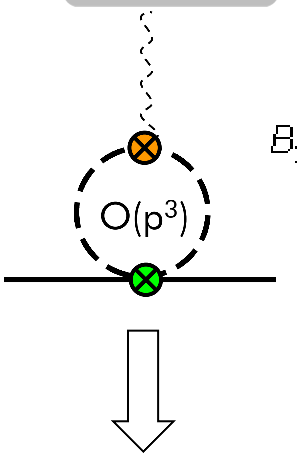
J. Zanotti et al. [QCDSF], PoS Lat2009

B_{20} and the anomalous gravitomagnetic moment

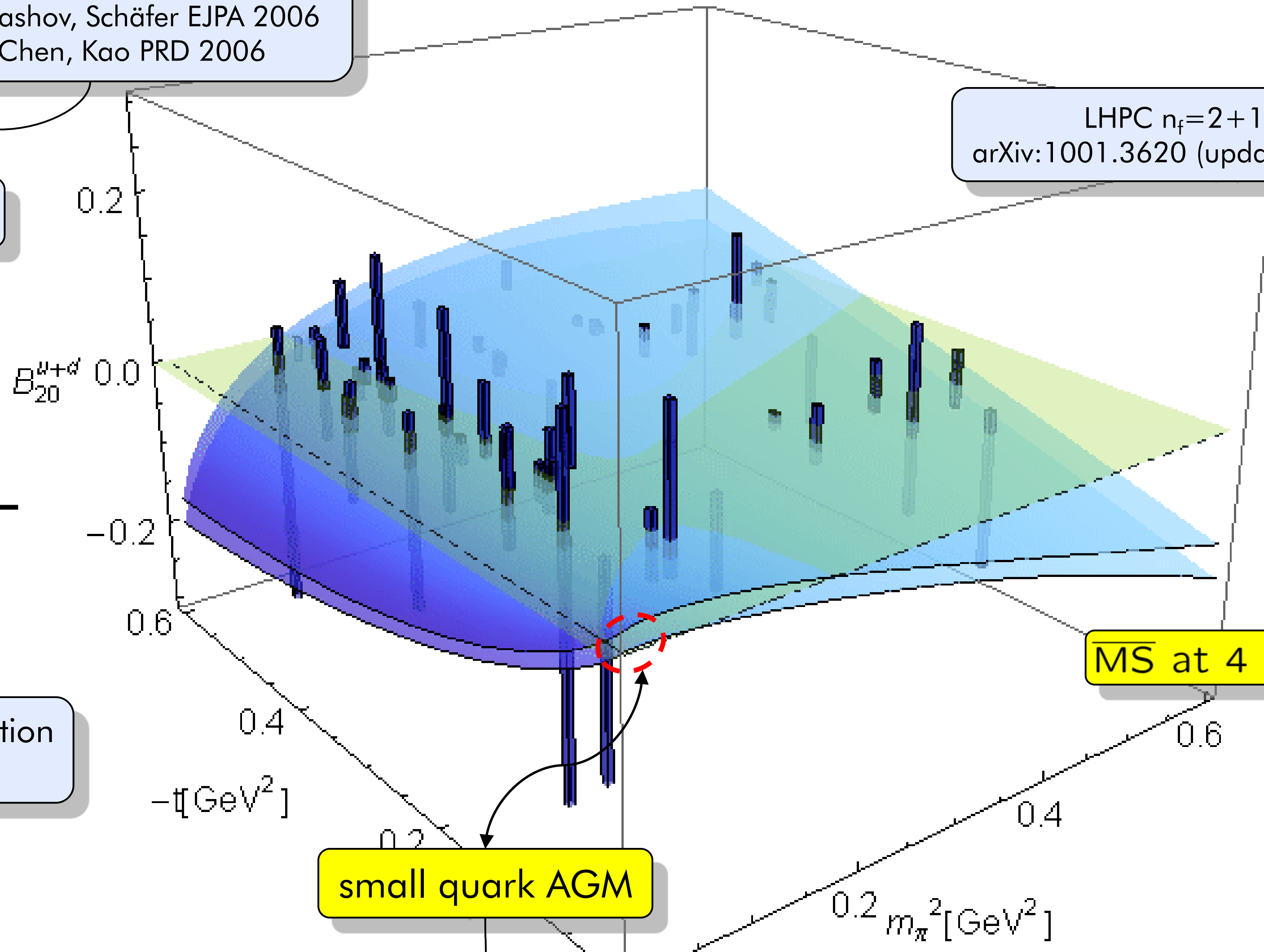
[Teryaev `99-; Brodsky, Hwang et al. `00-]

based on HBChPT by
Diehl, Manashov, Schäfer EJPA 2006
Ando, Chen, Kao PRD 2006

including



non-linear correlation in t and m□



LHPC n_f=2+1 mixed
arXiv:1001.3620 (updating PRD 2008)

$\overline{\text{MS}}$ at 4 GeV²

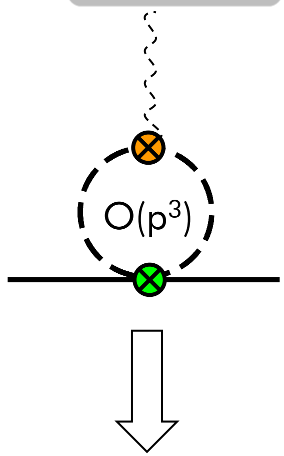
small quark AGM

$B_{20}^{u+d}(t=0, m_{\pi,\text{phys}}) = 0.024(15)$ → $B_{20}^{g}(t=0) \approx 0$!?

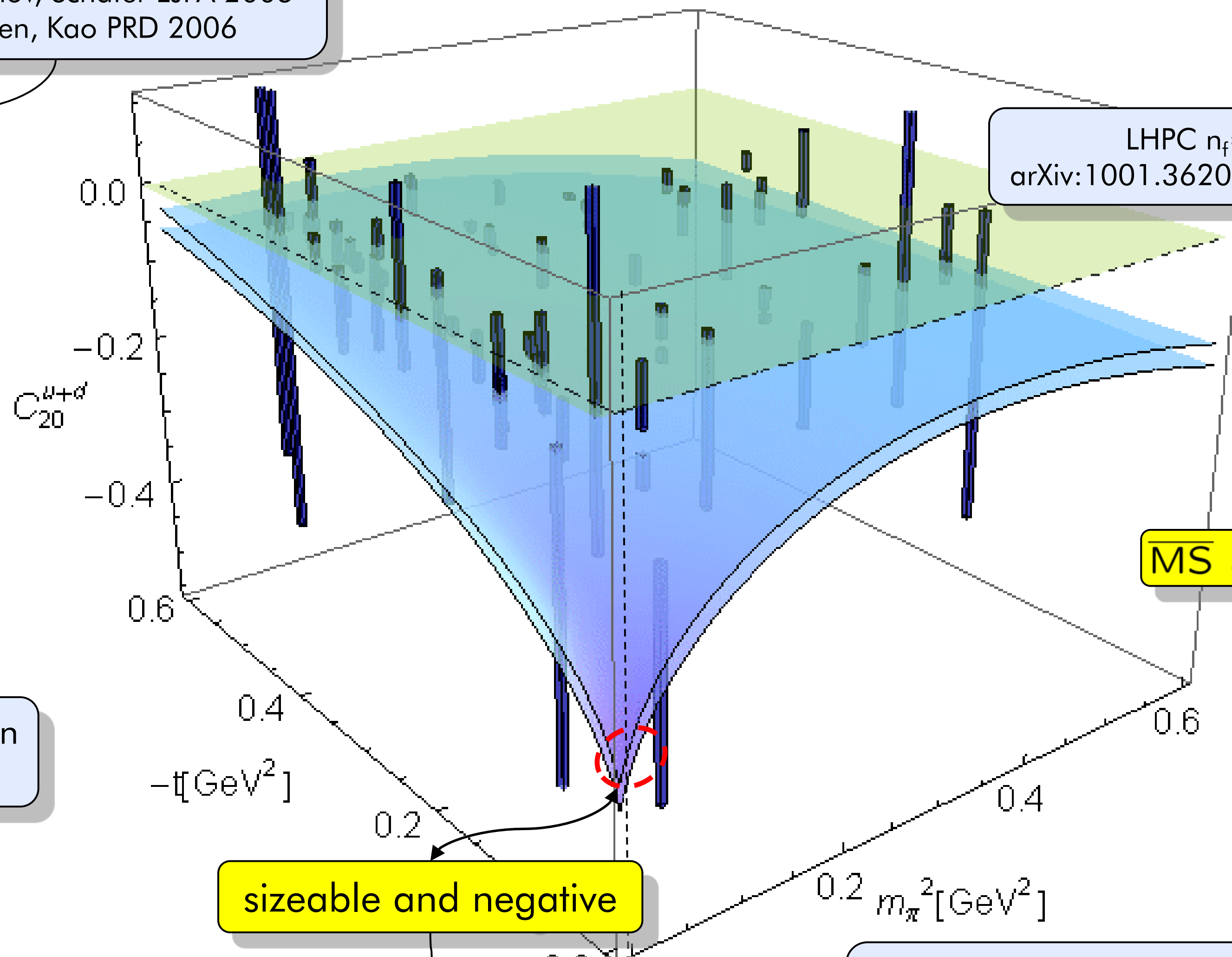
C_{20} and the second moment of the D-term [Polyakov&Weiss `99]

based on HBChPT by
Diehl, Manashov, Schäfer EJPA 2006
Ando, Chen, Kao PRD 2006

including



non-linear correlation in t and m



LHPC n_f=2+1 mixed
arXiv:1001.3620 (updating PRD 2008)

$\overline{\text{MS}}$ at 4 GeV^2

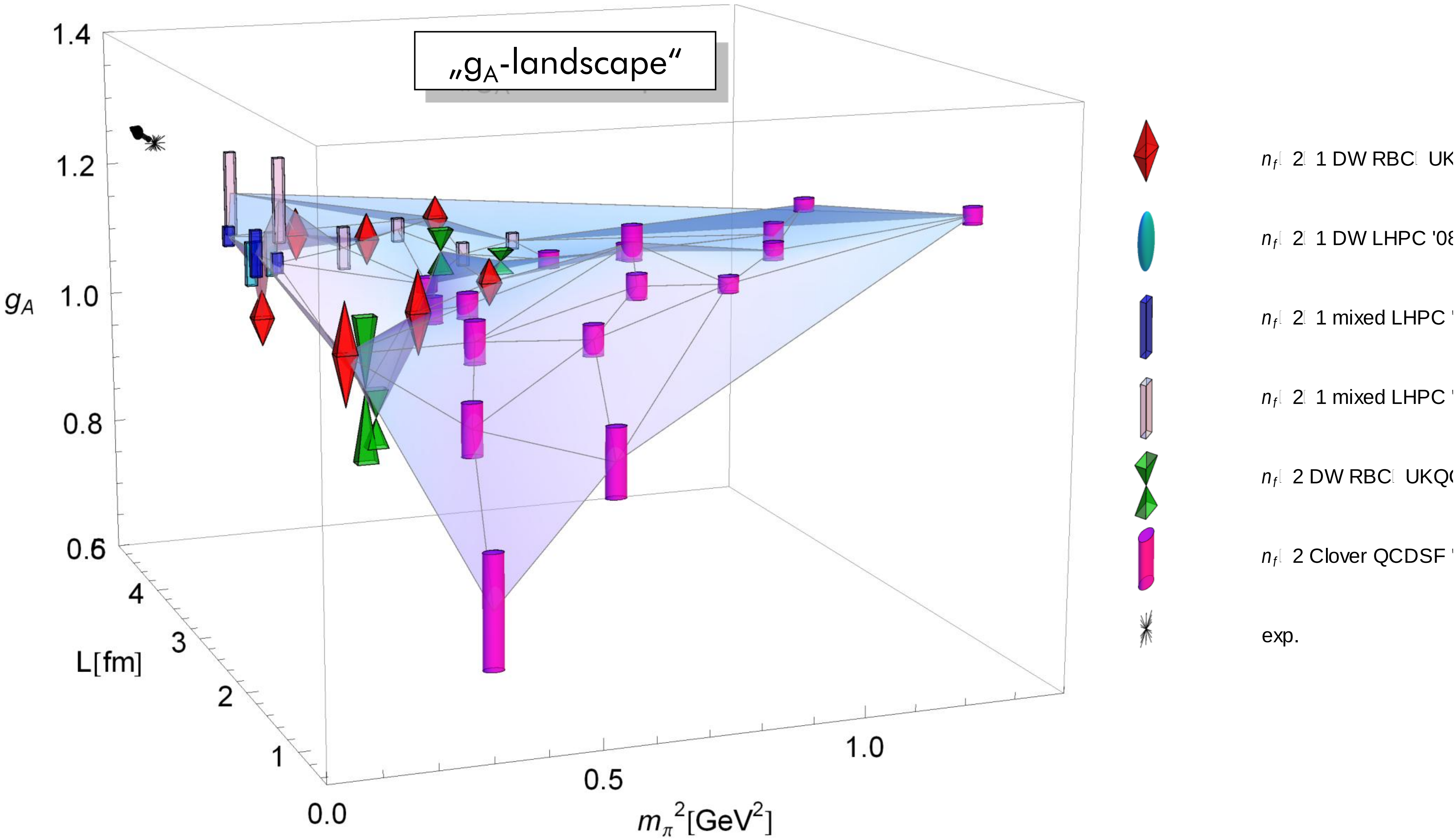
sizeable and negative

$$C_{20}^{u+d}(t=0, m_{\pi,\text{phys}}) = -0.33(2) \rightarrow \int_{-1}^{1} dz z D^{u+d}(z, t=0) \approx -1.3$$

!?

Nucleon axial vector coupling constant published data

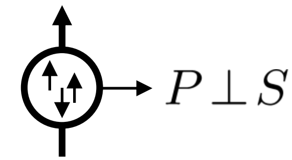
$$\langle P|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|P\rangle = g_A \bar{U}(P)\gamma_\mu\gamma_5 U(P)$$



Transversely polarized quarks in transversely polarized nucleons

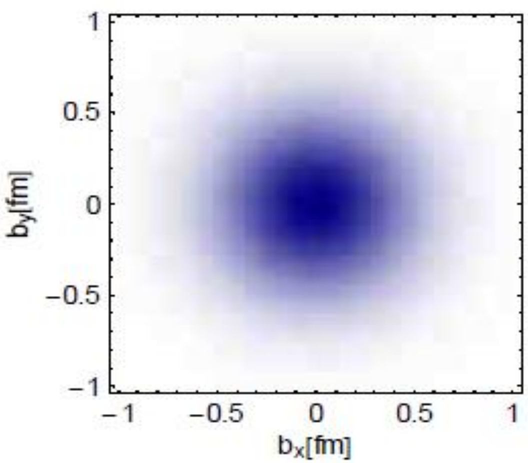
probability density
for (transversely polarized) quarks in
a (transversely polarized) proton

multipole-expansion

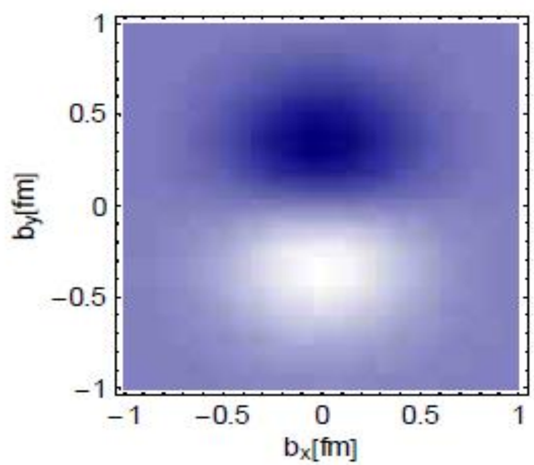


Diehl / PhH EPJC 2005

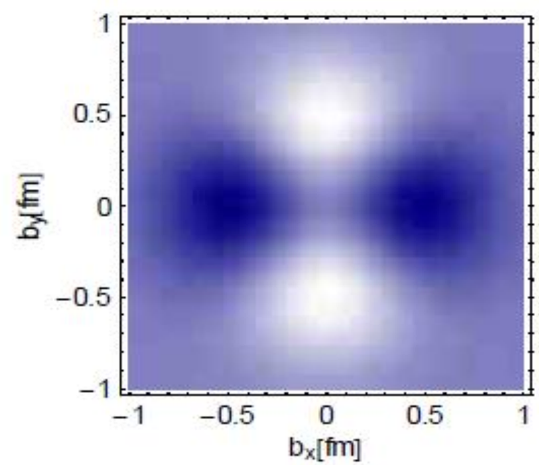
$$\langle P^+, 0_\perp, S_\perp | \hat{\rho}_T(x, b_\perp; s_\perp) | P^+, 0_\perp, S_\perp \rangle$$
$$= \frac{1}{2}\Big\{H + s^i_\perp S^i_\perp \Big(H_T - \frac{1}{4m^2}\Delta_b \widetilde{H}_T\Big) - \epsilon_{ij} S^i_\perp b^i_\perp \frac{1}{m} E' - \epsilon_{ij} s^i_\perp b^i_\perp \frac{1}{m} \overline{E}'_T + s^i_\perp (2 b^i_\perp b^j_\perp - b^2_\perp \delta^{ij}) S^j_\perp \frac{1}{m^2} \widetilde{H}''_T \Big\}$$



monopole

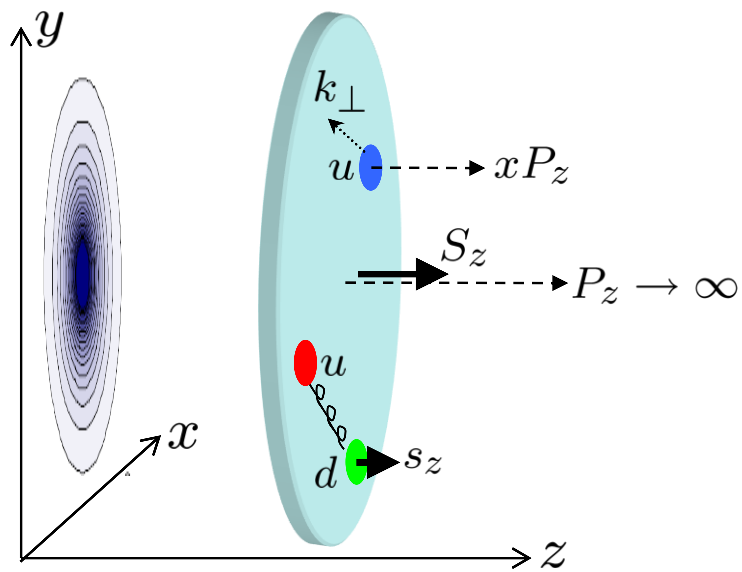


dipole



quadrupole

Intrinsic transverse momentum densities of the nucleon



$$\rho_L(x, \boldsymbol{k}_\perp; \Lambda, \boldsymbol{S}_\perp, \lambda) = \frac{1}{2}\left(f_1 + \lambda\Lambda g_1 + \left[\frac{\boldsymbol{S}_j \epsilon_{ji} \boldsymbol{k}_i}{m_N} f_{1T}^\perp\right] + \lambda \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{m_N} g_{1T}\right)$$

Diehl, PhH
EPJC 44 (2005)

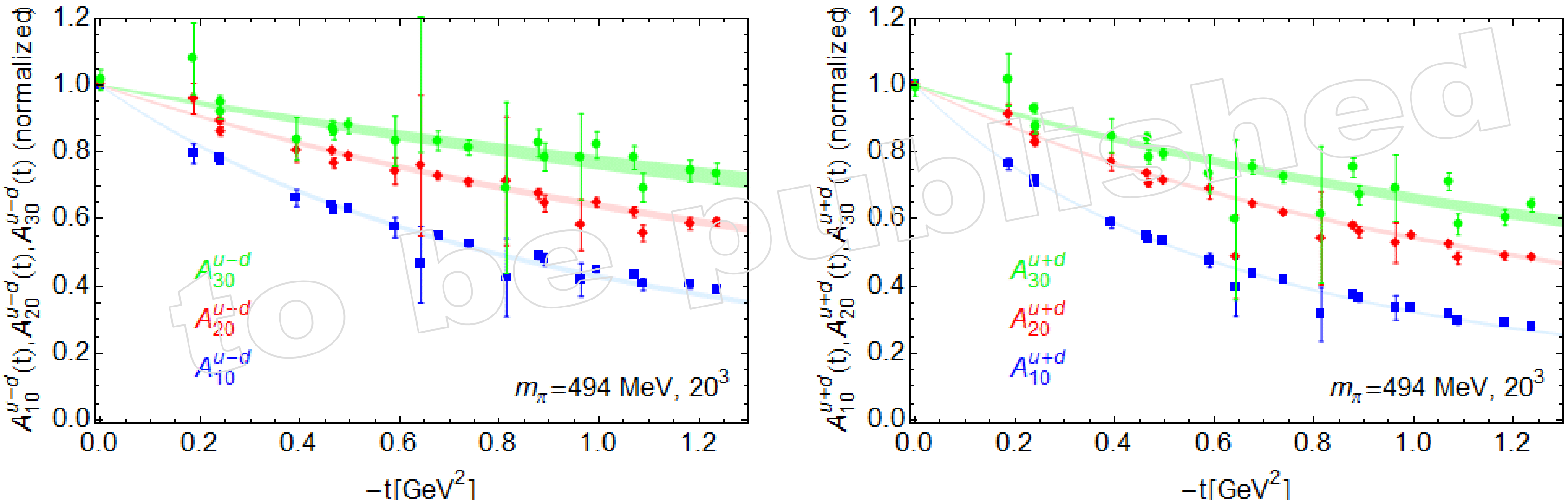
$$\rho_T(x, \boldsymbol{k}_\perp; \Lambda, \boldsymbol{S}_\perp, \boldsymbol{s}_\perp) = \frac{1}{2}\left(f_1 + \boldsymbol{s}_\perp \cdot \boldsymbol{S}_\perp h_1 + \left[\frac{\boldsymbol{s}_j \epsilon_{ji} \boldsymbol{k}_i}{m_N} h_1^\perp\right]\right.$$
$$\left. + \Lambda \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{s}_\perp}{m_N} h_{1L}^\perp + \frac{\boldsymbol{s}_j (2\boldsymbol{k}_j \boldsymbol{k}_i - \boldsymbol{k}_\perp^2 \delta_{ji}) \boldsymbol{S}_i}{2m_N^2} h_{1T}^\perp\right)$$

Boglione, Mulders PRD 60 (1999)

correlations in x and t

$\bar{x} \to 1 \Leftrightarrow$

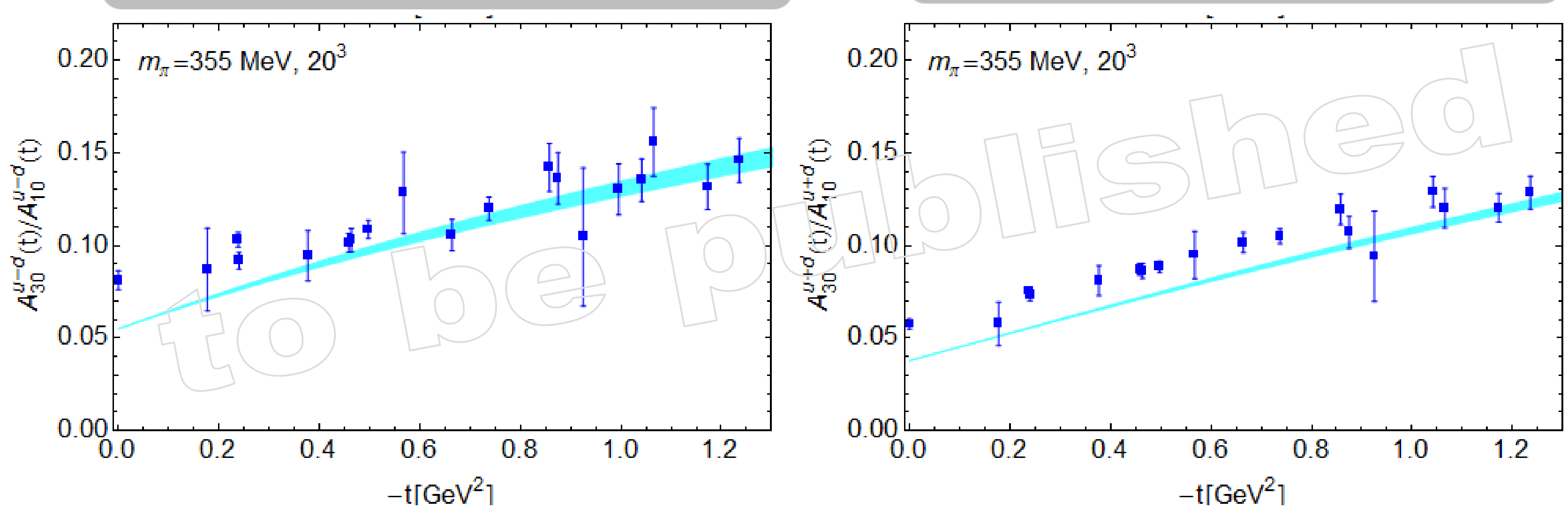
$n \to \infty$



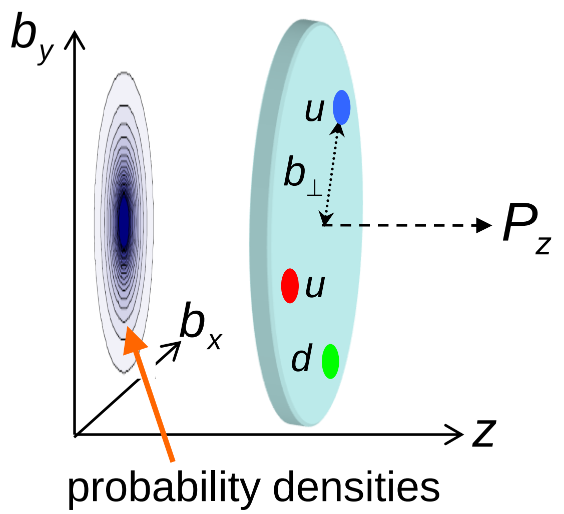
compared to parametrization/ansatz by Diehl, Feldmann, Jakob, Kroll EPJC 2005

$$H_v^q(x,t) = q_v(x)\exp\left[t f_q(x)\right]$$

$$\frac{A_{30}^{u\pm d}(t)}{A_{10}^{u\pm d}(t)}$$



Nucleon form factors



Dirac and Pauli FFs → $\langle P'|\bar{q}\gamma_\mu q|P\rangle = \bar{U}(P')\left\{\gamma_\mu F_1(t) + \frac{i\sigma_{\mu\nu}\Delta^\nu}{2m}F_2(t)\right\}U(P)$

electric and magnetic FFs →

$$G_E(t) = F_1(t) + \frac{t}{4m_N^2}F_2(t)$$

$$G_M(t) = F_1(t) + F_2(t) \qquad \text{where } t \mathrel{\hat{=}} -Q^2$$

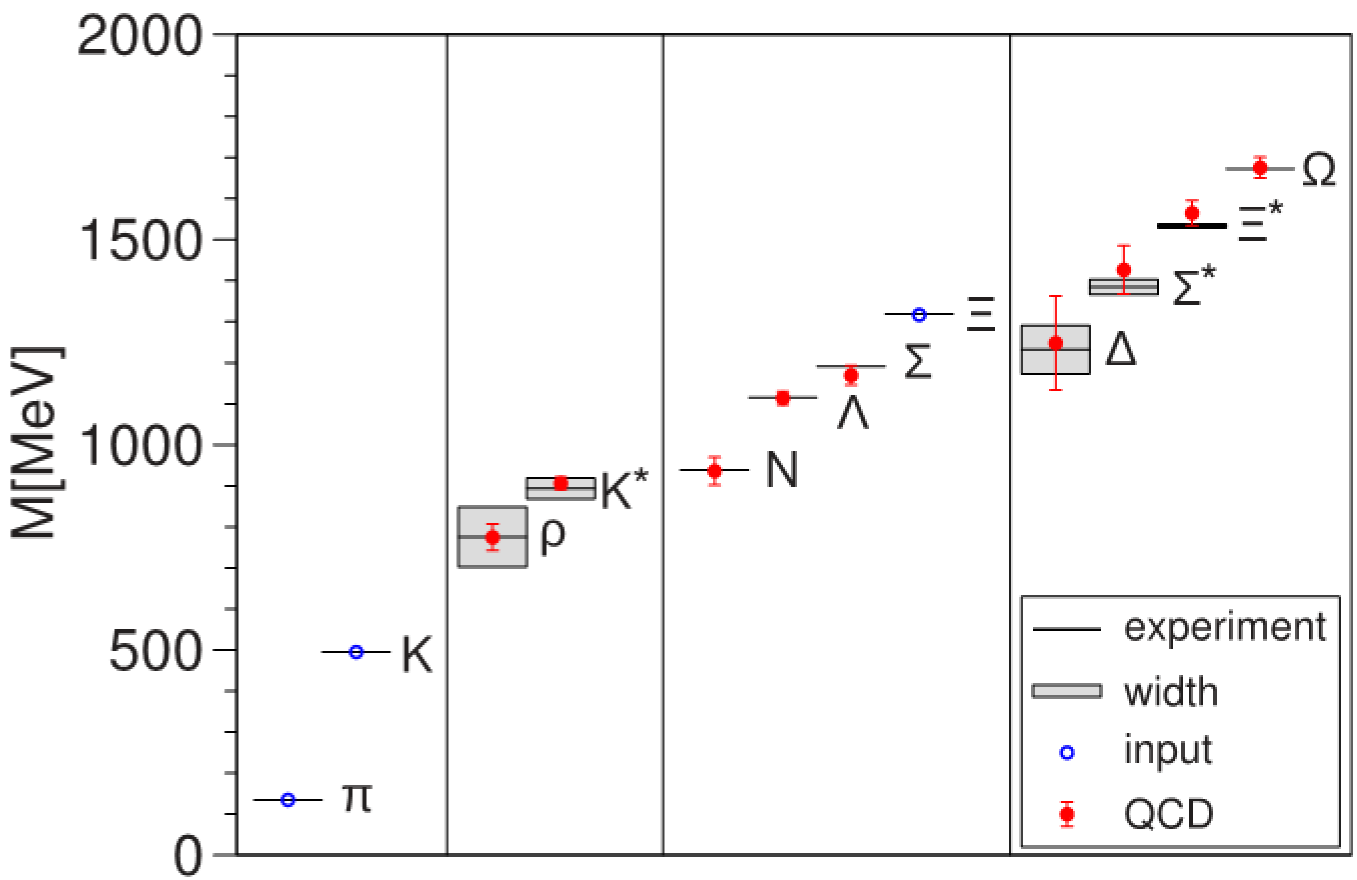
remember →

$$\underbrace{\langle p|\sum_{q=u,d} e_q\bar{q}\gamma_\mu q|p\rangle}_{G_E^p,G_M^p} - \underbrace{\langle n|\sum_{q=u,d} e_q\bar{q}\gamma_\mu q|n\rangle}_{G_E^n,G_M^n} = \underbrace{\langle p|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|p\rangle}_{G_E^{u-d},G_M^{u-d}} \mathrel{\hat{=}} \text{isovector}$$

(anomalous) magnetic moment → $\mu = G_M(t=0) = F_1(t=0) + \kappa$

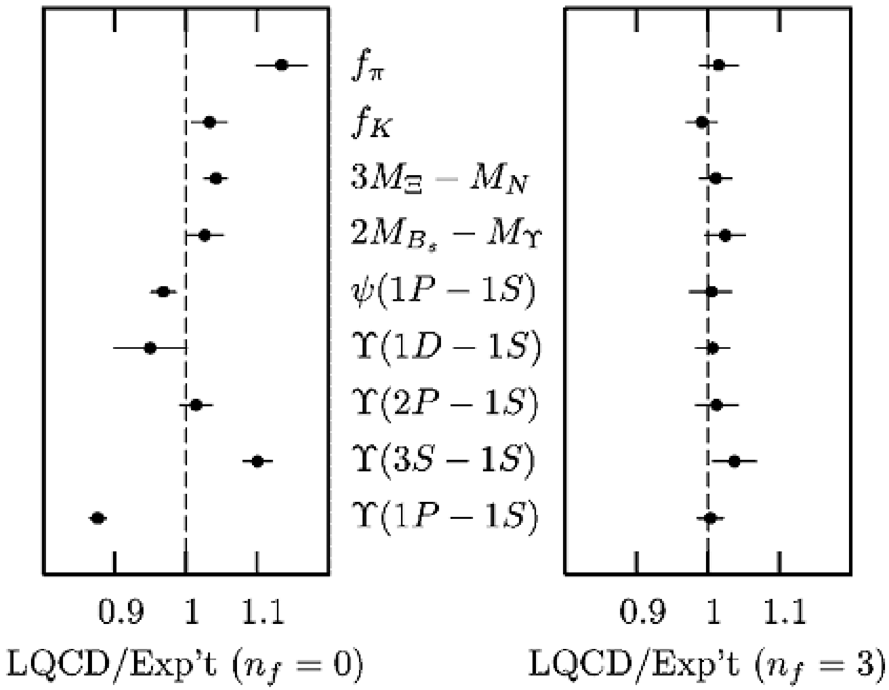
charge radii → $\langle r_1^2\rangle = -\frac{6}{F_1(0)}\frac{d}{dt}F_1(t)\Big|_{t=0}$ — e.g. → $\langle r_1^2\rangle_p^{u-d} = \langle r_E^2\rangle_p - \langle r_E^2\rangle_n - \frac{3}{2m_N^2}\kappa_{p-n}$

Lattice QCD propaganda

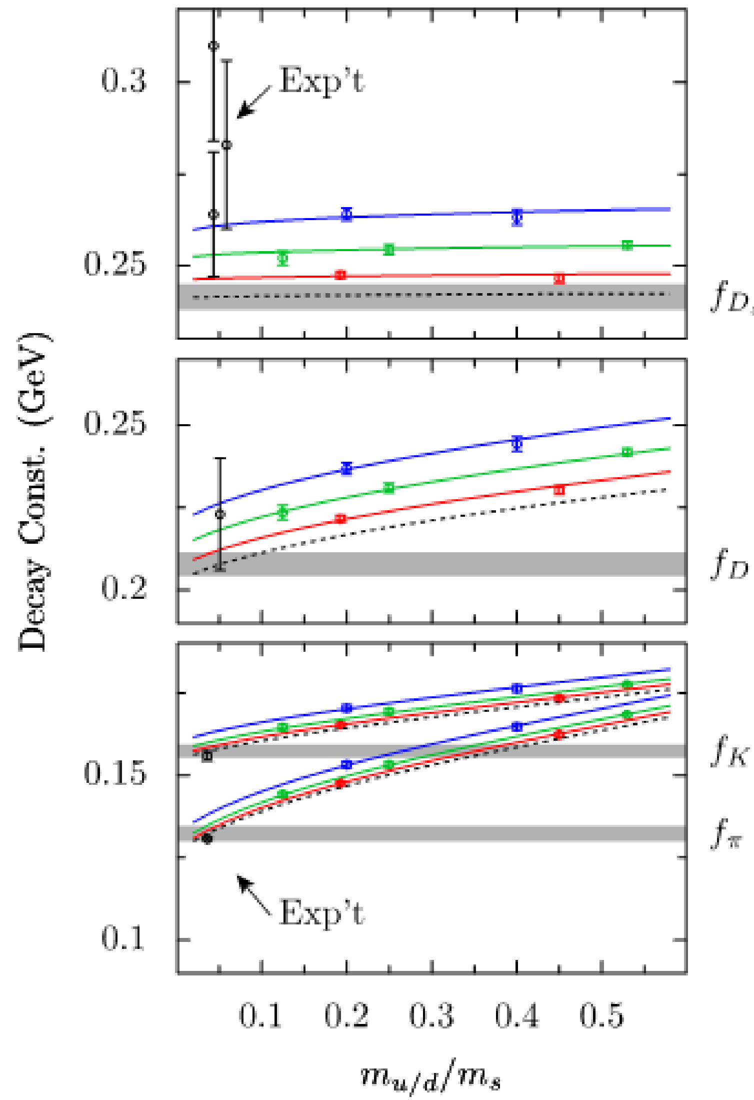


BMW (Dürr et al.) Nature 2009

Lattice QCD propaganda



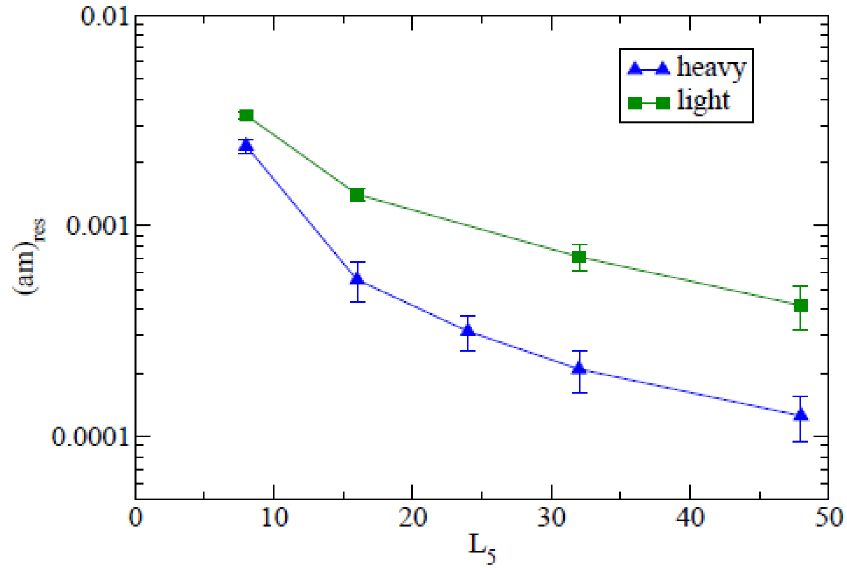
Davies, Lepage et al. PRL 2004



Davies, Lepage et al. PRL 2008

Lattice parameters – LHPC

- domain-wall-fermions on a staggered "Asqtad" staggered sea ("hybrid" formalism) with HYP-smearing
- use of staggered quarks is "a matter of taste"
- $N_f = 2+1$, but only connected contributions
- $Ls = 16$, $m_{res} \le 0.1 m_q$
- inverse lattice-spacing is $a^{-1} \approx 1.6$ GeV
- pion masses as low as 300MeV in volumes $\le (3.5\text{fm})^3$
- one projector $\tilde{\Gamma}_{pol} = \frac{1}{4}(1+\gamma_0)(1-\gamma_5\gamma_3)$
- two sink-momenta $p' = (0,0,0), (-1,0,0)$



operator renormalization: $Z_{\mathcal{O}} = \frac{Z_{\mathcal{O}}^{pert}}{Z_A^{pert}} Z_A^{nonpert}$ → $\overline{\text{MS}}$ at 4 GeV^2

of „measurements" increased by factor 8 compared to PRD 77 094502 (2008)

dataset	Ω	#	$(am)_q^{\text{Asqtad}}$	$(am)_q^{\text{DWF}}$	$(am)_\pi^{\text{Asqtad}}$	$(am)_\pi^{\text{DWF}}$	$(am)_N^{\text{Asqtad}}$	$(am)_N^{\text{DWF}}$	m_π^{DWF} [MeV]
1	$20^3 \times 32$	425	0.050/0.050	0.0810	0.4836(2)	0.4773(9)	1.057(5)	0.986(5)	758.9(1.4)
2		350	0.040/0.050	0.0478	0.4340(3)	0.4293(10)	1.003(3)	0.938(8)	682.6(1.6)
3		564	0.030/0.050	0.0644	0.3774(2)	0.3747(10)	0.930(3)	0.869(6)	595.8(1.6)
4		486	0.020/0.050	0.0313	0.3109(2)	0.3121(11)	0.854(3)	0.814(7)	496.2(1.7)
5		655	0.010/0.050	0.0138	0.2242(2)	0.2243(10)	0.779(6)	0.730(12)	356.6(1.6)
6	$28^3 \times 32$	270	0.010/0.050	0.0138		0.2220(9)		0.766(15)	352.3(1.4)
7	20^3x32	460	0.007/0.05			0.1842(7)			292

Form factors of the energy momentum tensor and fundamental sumrules

$\langle P'|T^{\mu\nu}|P\rangle$ ↔ graviton-coupling spin-2 coupling

$$\langle P'|T^{\mu\nu}|P\rangle = \overline{U}(P')\Big\{\gamma^{\mu}\overline{P}^{\mu}A_{20}(\Delta^2) + \frac{i\sigma^{\mu\rho}\Delta_{\rho}\overline{P}^{\nu}}{2m_N}B_{20}(\Delta^2) + \frac{\Delta^{\mu}\Delta^{\nu}}{m_N}C_{20}(\Delta^2)\Big\}U(P)$$

translation invariance

rotational symmetry

conservation of momentum

conservation of angular momentum

momentum sumrule

$$1 = \sum_q A^q_{20}(0) + A^g_{20}(0)$$
$$= \sum_q \langle x\rangle_q + \langle x\rangle_g$$

vanishing of the anomalous gravitomagnetic moment

$$0 = \sum_q B^q_{20}(0) + B^g_{20}(0)$$

Ji's nucleon spin sum rule

$$\frac{1}{2} = S_z = \frac{1}{2}\Big(\sum_q A^q_{20}(0) + A^g_{20}(0) + \sum_q B^q_{20}(0) + B^g_{20}(0)\Big)$$
$$= \sum_q J_q + J_g = \sum_q \frac{1}{2}\Delta\Sigma_q + \sum_q L_q + J_g$$

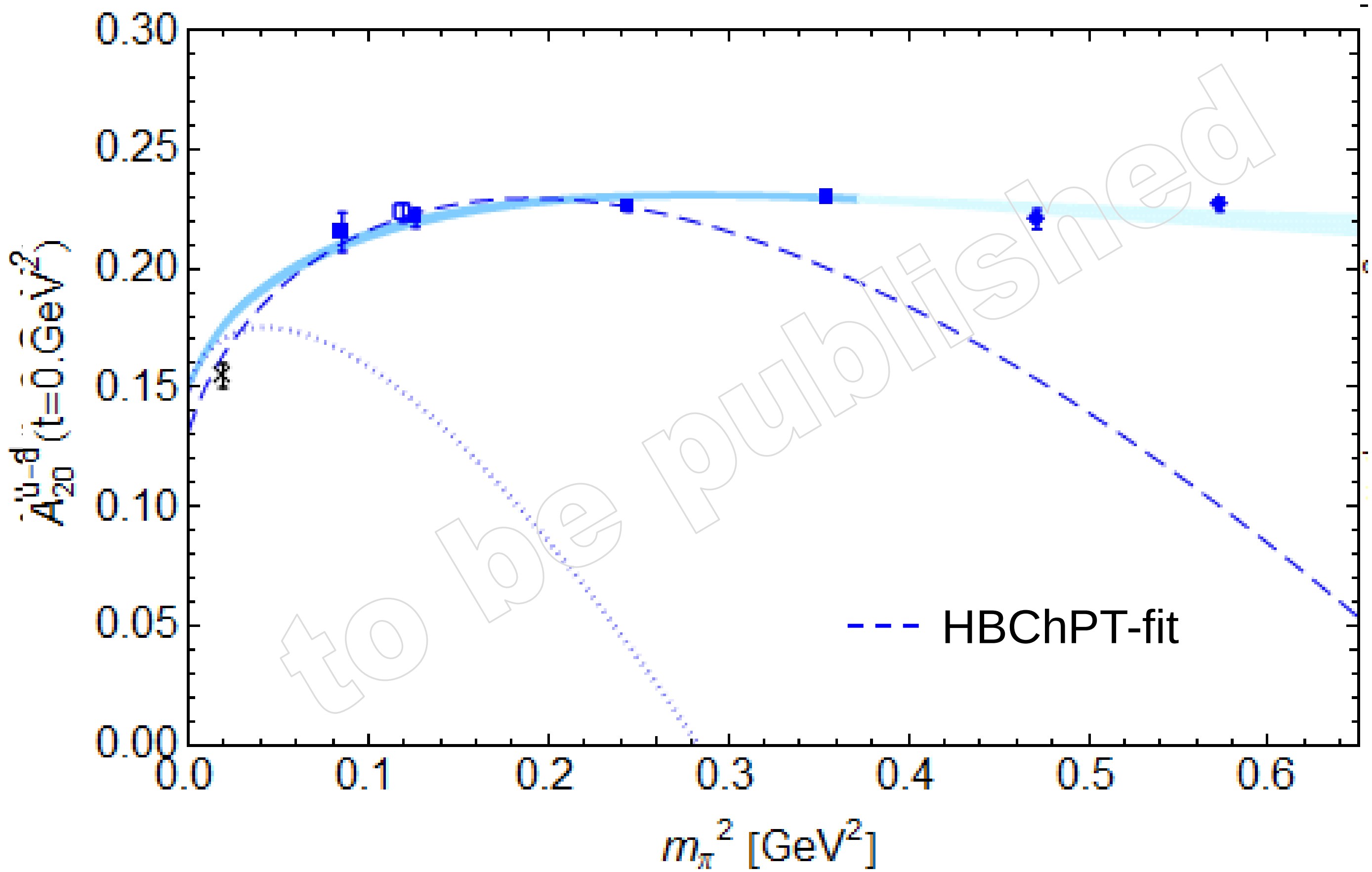
$$L_q \equiv J_q - \frac{1}{2}\Delta\Sigma_q$$
$$(L_g \equiv J_g - \Delta G)$$

everything is: -gauge-invariant
-scale and scheme dependent
-measurable

Form factors of the energy momentum tensor

isovector quark momentum fraction

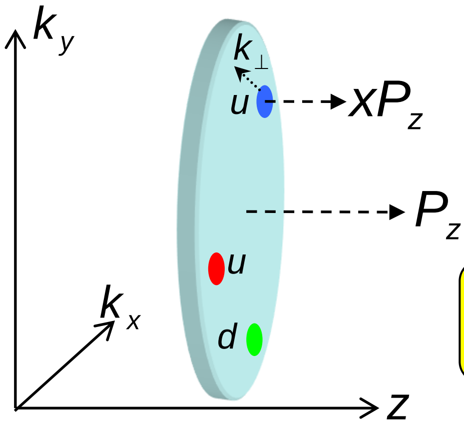
compared to LHPC PRD 77 094502 (2008)



TMDs in lattice QCD

PhH, B. Musch, J. Negele, A. Schäfer, arXiv:0908.1283
B. Musch, PhD thesis arXiv:0907.2381

Transverse momentum dependent PDFs - formalism

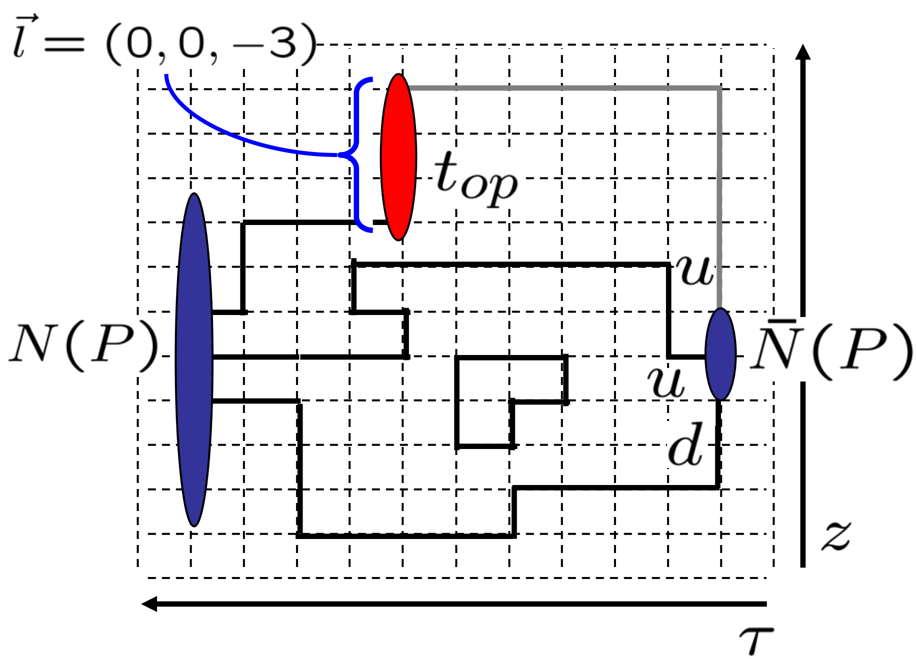


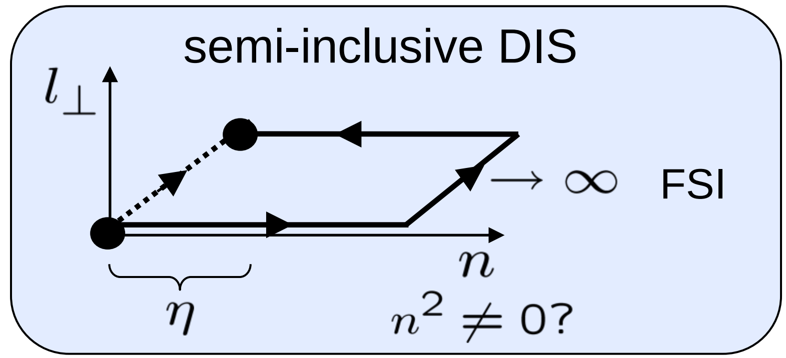
$$f(x,k_\perp) \propto \int d\eta\, d^2 l_\perp\, e^{ix\eta P} e^{-il_\perp \cdot k_\perp} \langle P | \bar{q}(-\tfrac{\eta}{2} n, l_\perp) \Gamma U q(\tfrac{\eta}{2} n, 0_\perp) | P \rangle$$

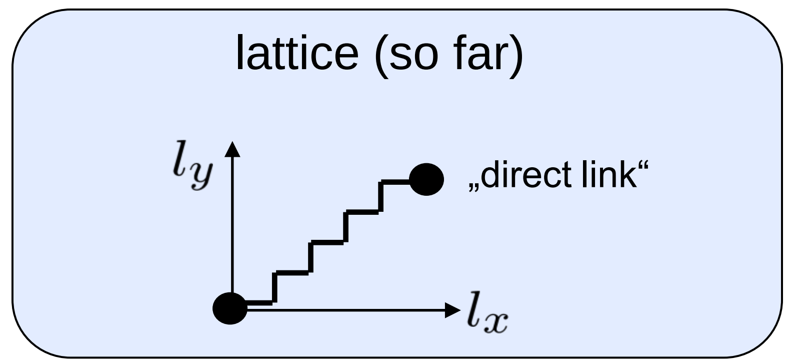
$$f(x,k_\perp) \propto \int d(l\cdot P)\, d(l^2)\, e^{ix(l\cdot P)} J_0(\sqrt{-l^2}\,|k_\perp|)\, A_2(l\cdot P, l^2)$$

complex amplitude A_2

choice of Wilson line U







Transverse momentum dependent PDFs - formalism

$$\langle P|\overline{q}(-\frac{\eta}{2}n, l_\perp)\Gamma U q(\frac{\eta}{2}n, 0_\perp)|P\rangle$$

$$\widetilde{A}_2(l^2, l\cdot P), \widetilde{A}_3, \widetilde{A}_6, \widetilde{A}_7, \widetilde{A}_8, \widetilde{A}_{9m}, \widetilde{A}_{10}, \widetilde{A}_{11}$$

Mulders, Tangermann NPB 1996
Boer, Mulders PRD 57 (1998)

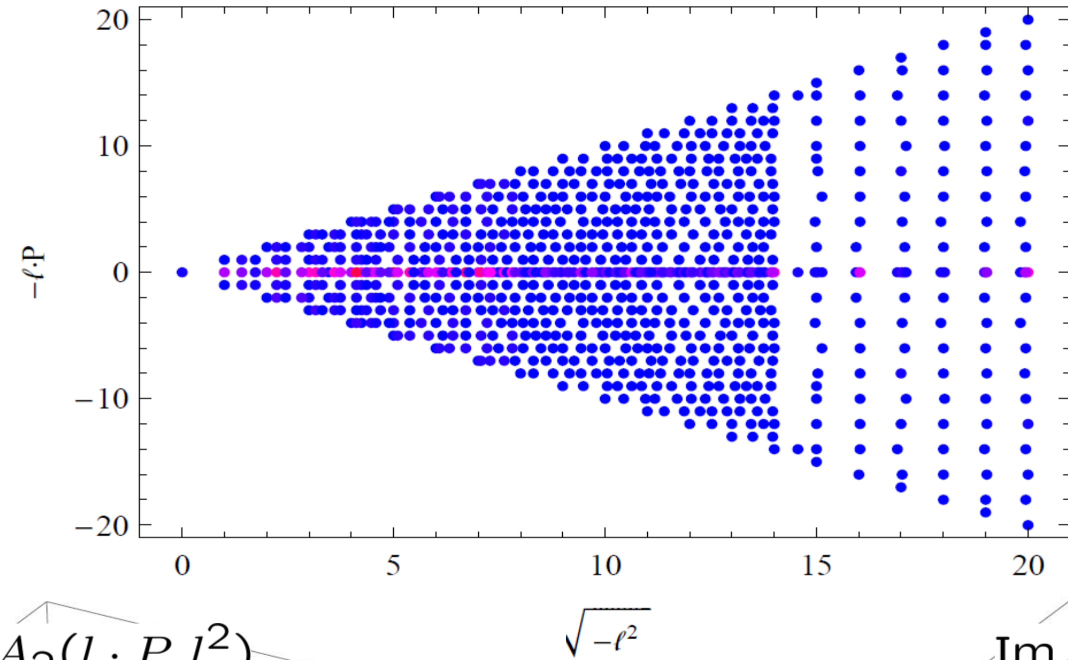
Fourier-transformation

$$l^2 \leftrightarrow k_\perp^2,\ l\cdot P \leftrightarrow x$$

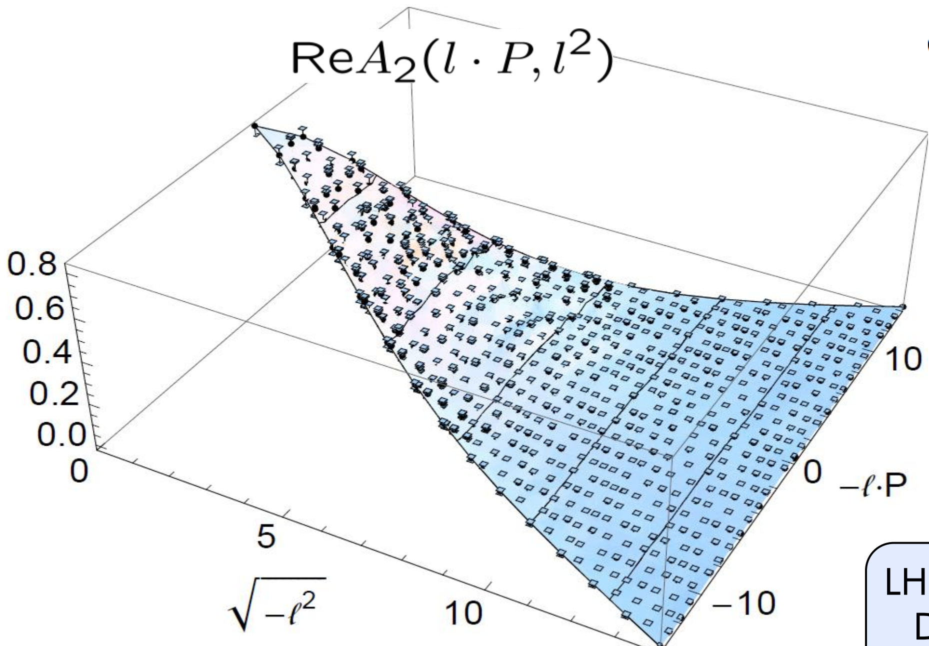
$$l\cdot P = 0 \leftrightarrow \int_{-1}^{1} dx$$

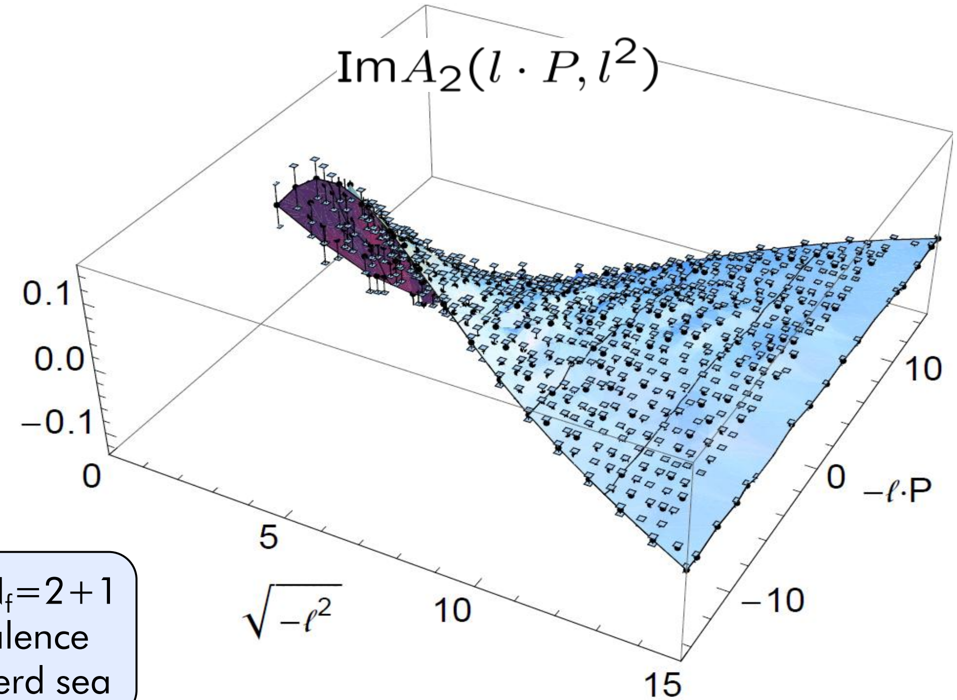
$$\begin{aligned} \widetilde{A}_2 &\leftrightarrow f_1(x, k_\perp^2), \\ \widetilde{A}_{6,7} &\leftrightarrow g_1,\ \widetilde{A}_7 \leftrightarrow g_{1T} \\ \widetilde{A}_{9m} &\leftrightarrow h_1,\ \widetilde{A}_{10,11} \leftrightarrow h_{1L}^\perp,\ \widetilde{A}_{11} \leftrightarrow h_{1T}^\perp \end{aligned}$$

Overview of numerical results for A_2



all for m_π ~ 500 MeV





LHPC, $N_f=2+1$
DW-valence
+staggerd sea

Renormalization

potential power-divergence

$$U[C_l] \propto e^{-\delta m\, l} = e^{-\frac{\widehat{\delta m}}{a}\, l}$$

$$V_{\bar{Q}Q}(R) = \lim_{T\to\infty} \partial_T \ln\langle W(R,T)\rangle = V^{\text{ren}}_{\bar{Q}Q}(R) + 2\delta m$$

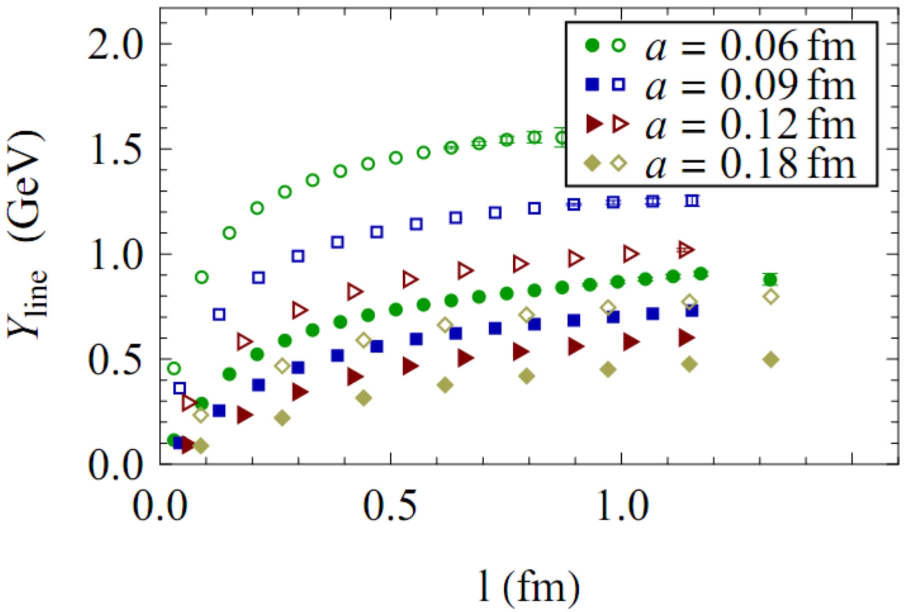
renormalization condition

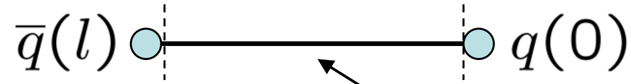
$$V_{string}(R) = \sigma R - \frac{\pi}{12R} + C^{\text{ren}}$$

$$C^{\text{ren}} = 0$$

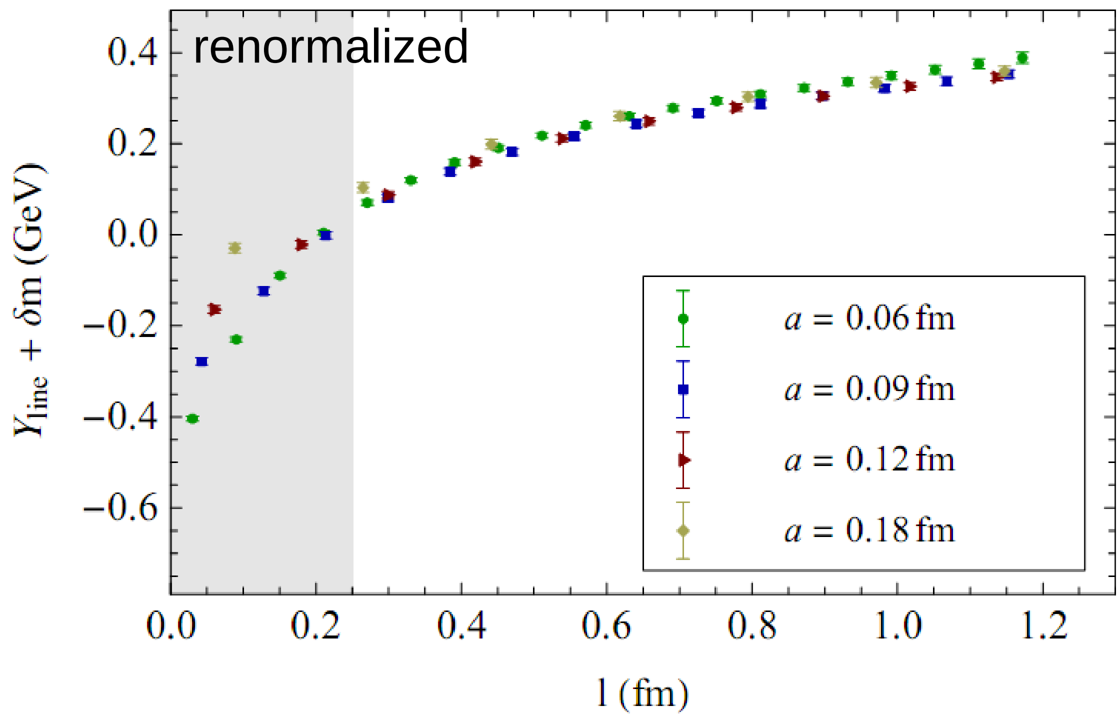
a[fm]	$\widehat{\delta m}$
0.12fm	0.1553(47)
0.08fm	0.1639(35)
0.06fm	0.1578(17)

Illustration of renormalization





$$Y_{\rm line}(l) = -\frac{d}{dl} \ln \mathrm{Tr}\langle U[C_l]\rangle$$



„Regularization“ and multiplicative renormalization

Gaussian parametrization of the invariant amplitudes — $2\widetilde{A}_i(l^2, l\cdot P{=}0) = c_i e^{l^2/\sigma_i^2}$

restricted to l ~ 0.25 … 2 fm — reg. *potential* divergences at small l ★ large k_γ

renormalize multiplicatively such that $\widetilde{A}_2^{u-d,ren}(l^2{=}0, l\cdot P{=}0) = F_1^{u-d}(t{=}0) = 1$

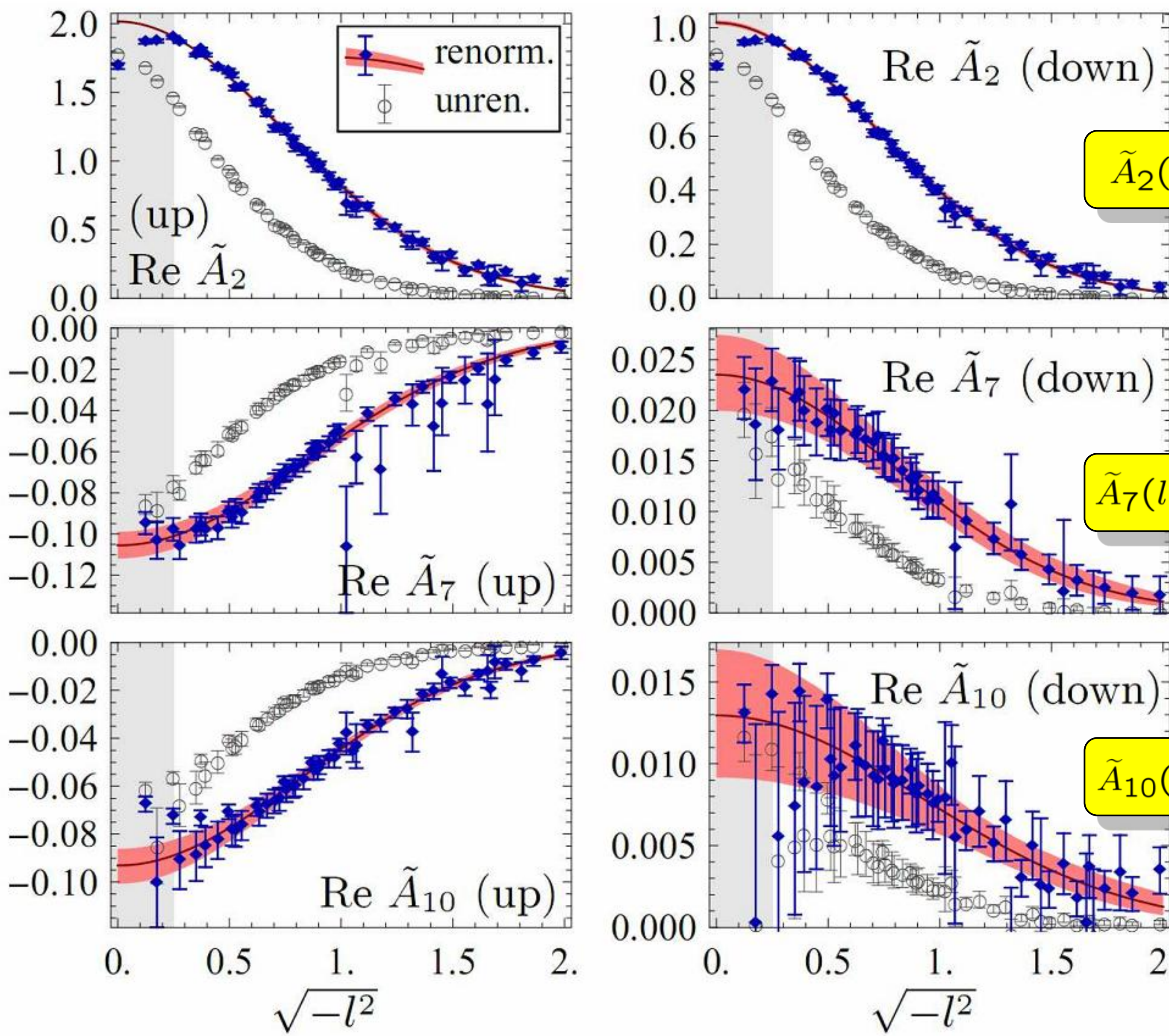
	c		$2/\sigma$ (GeV)
$\tilde{A}_2^u$	2.0159(86)	$= f_{1,u}^{(0,0)}$	0.3741(72)
$\tilde{A}_2^d$	1.0192(90)	$= f_{1,d}^{(0,0)}$	0.3839(78)
$\tilde{A}_6^u$	-0.920(35)	$= -g_{1,u}^{(0,0)}$	0.311(11)
$\tilde{A}_6^d$	0.291(19)	$= -g_{1,d}^{(0,0)}$	0.363(18)
$\tilde{A}_{9m}^u$	0.931(29)	$= h_{1,u}^{(0,0)}$	0.3184(90)
$\tilde{A}_{9m}^d$	-0.254(16)	$= h_{1,d}^{(0,0)}$	0.327(15)
$\tilde{A}_7^u$	-0.1055(66)	$= -g_{1T,u}^{(0,1)}$	0.328(14)
$\tilde{A}_7^d$	0.0235(38)	$= -g_{1T,d}^{(0,1)}$	0.346(36)
$\tilde{A}_{10}^u$	-0.0931(73)	$= h_{1L,u}^{\perp(0,1)}$	0.340(14)
$\tilde{A}_{10}^d$	0.0130(40)	$= h_{1L,d}^{\perp(0,1)}$	0.301(48)

$2/\sigma_i \widehat{=} \langle k_\perp^2\rangle^{1/2}$

$\rightarrow g_A = 1.209(36)$

at this stage, better not compare quantitatively with TMD-phenomenology (e.g. Anselmino et al.)

l^2-dependence of invariant amplitudes (renormalized)



PhH, B. Musch et al.
arXiv:0908.1283

$\tilde{A}_2(l \cdot P = 0, l^2) \leftrightarrow f_1^{(n=1)}(k_\perp^2)$

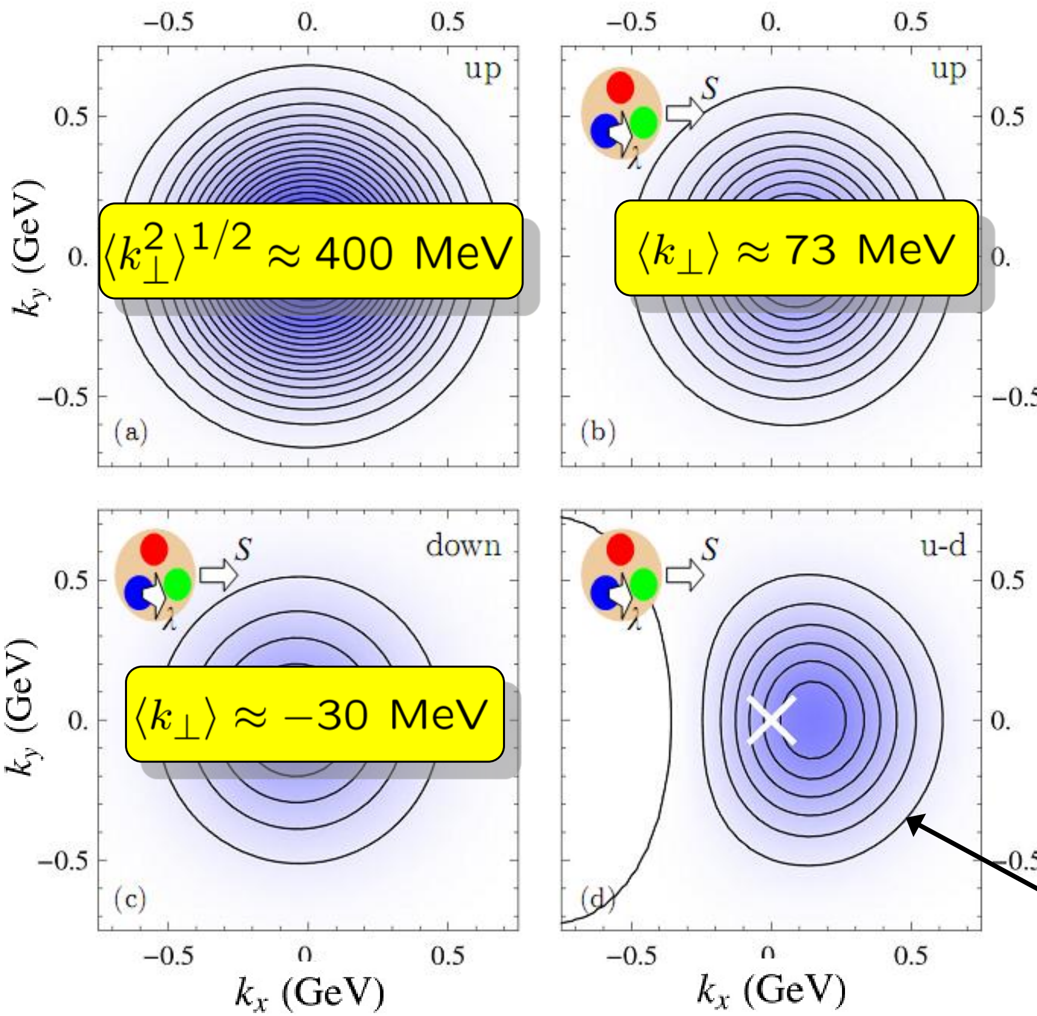
$\tilde{A}_7(l \cdot P = 0, l^2) \leftrightarrow -g_{1T}^{(n=1)}(k_\perp^2)$

$\tilde{A}_{10}(l \cdot P = 0, l^2) \leftrightarrow h_{1L}^{\perp(n=1)}(k_\perp^2)$

comp. to LC quark model:
Pasquini et al PRD 2008

Intrinsic transverse momentum densities of the nucleon

$$\rho(x, k_\perp; \lambda, S_\perp) = \frac{1}{2}\left(f_1(x, k_\perp^2) + \lambda \frac{k_\perp \cdot S_\perp}{m_N} g_{1T}(x, k_\perp^2)\right)$$



Musch et al. $n_f = 2+1$ mixed preliminary and PoSLC2008

~~$g_{1T}(x, k_\perp) \leftrightarrow$ GPDs~~

genuine effect of intrinsic transverse momentum of quarks

Approximate relations between GPDs and TMDs

T-odd Sivers

$$f_1 \leftrightarrow H, \qquad f_{1T}^{\perp} \leftrightarrow -E', \qquad g_1 \leftrightarrow \tilde{H},$$

$$h_1 \leftrightarrow H_T - \Delta_b \tilde{H}_T/(4m^2),$$

$$h_1^{\perp} \leftrightarrow -(E'_T + 2\tilde{H}'_T), \qquad h_{1T}^{\perp} \leftrightarrow 2\tilde{H}''_T.$$

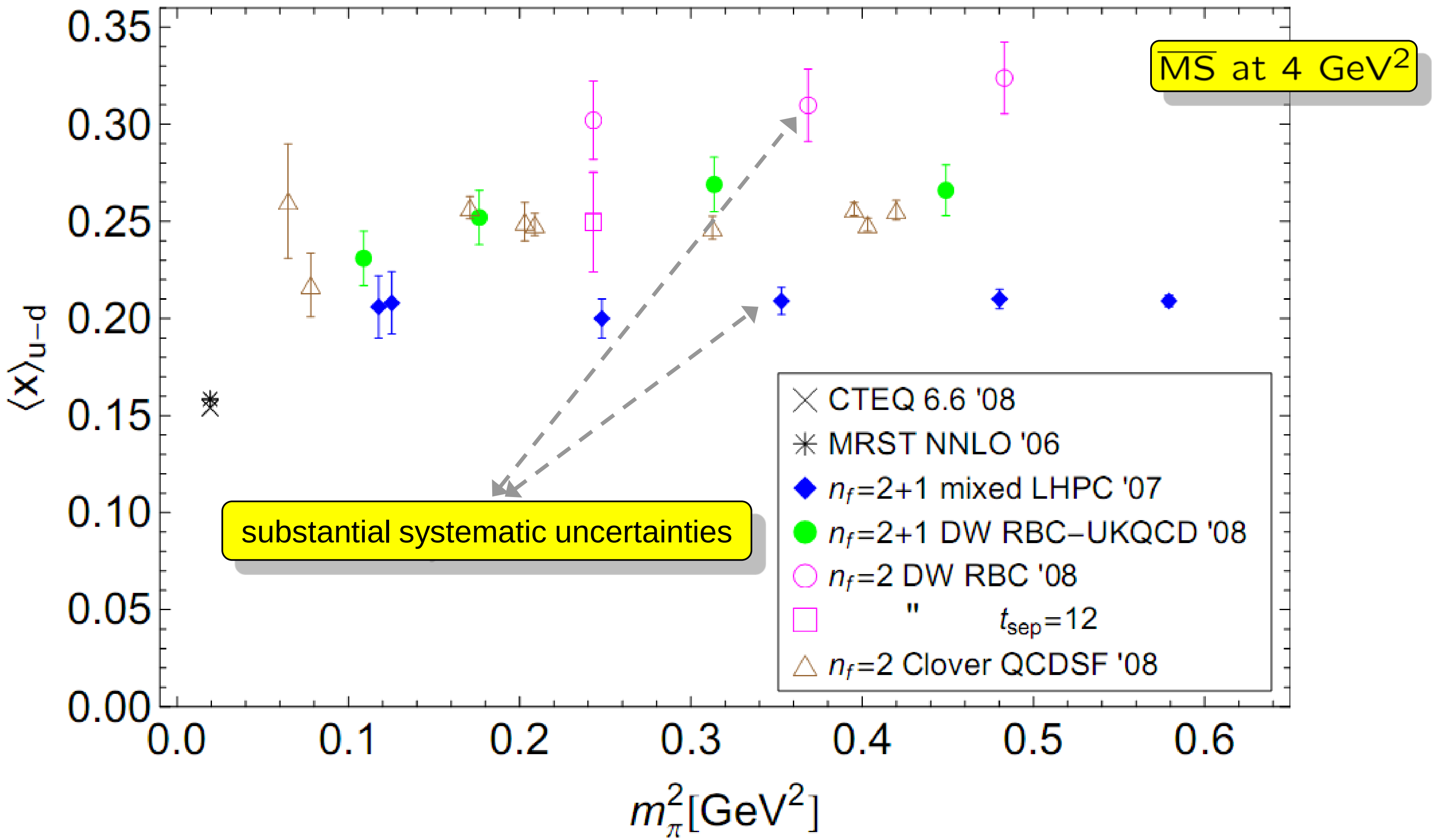
T-odd Boer-Mulders

Burkardt PRD 2002
Diehl, PhH EPJC 2005
Metz et al. 2007

Momentum fraction of quarks in the nucleon

$$\langle P|\overline{q}\gamma^{\{\mu}D^{\nu\}}q|P\rangle = \overline{U}(P)\gamma^{\{\mu}P^{\nu\}}U(P)\langle x\rangle$$

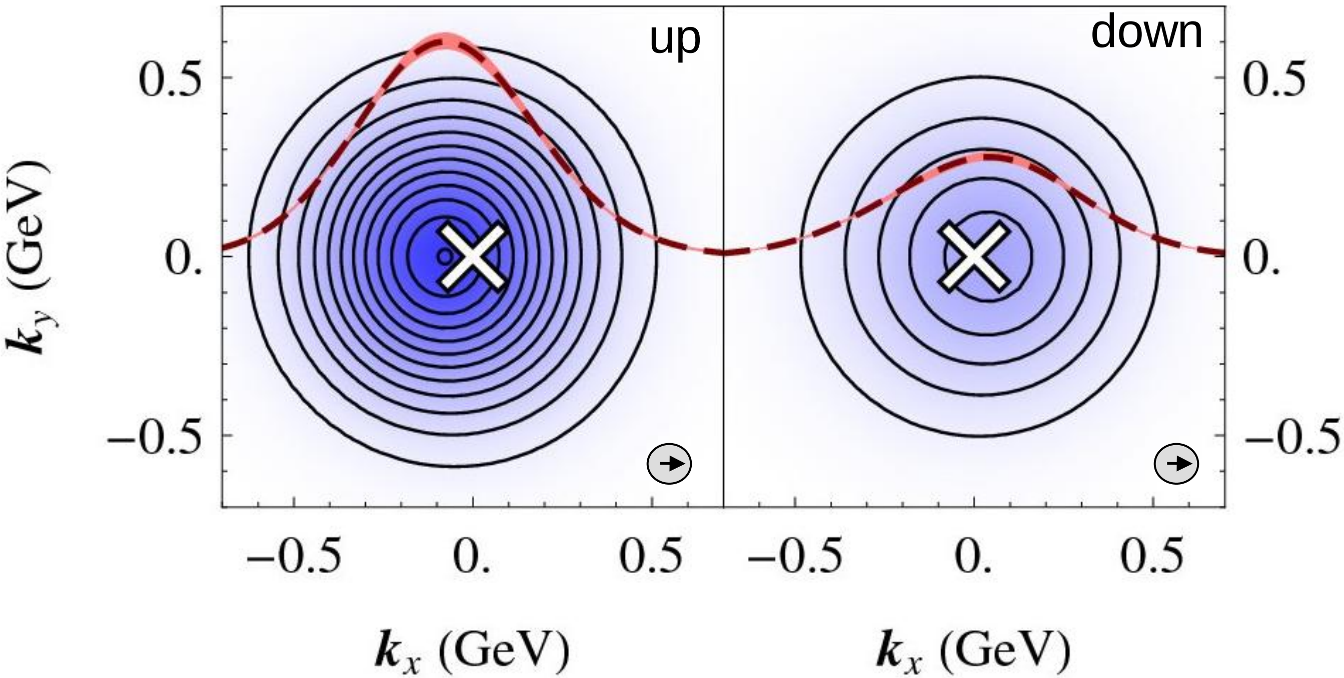
$$\langle x\rangle = A_{20}(0) = \int_{-1}^{+1} dx\, x\, q(x) = \langle x\rangle_q + \langle x\rangle_{\bar{q}}$$



Intrinsic transverse momentum densities of the nucleon

$$\rho(x, k_\perp; \Lambda, s_\perp) = \frac{1}{2}(f_1 + \Lambda \frac{k_\perp \cdot s_\perp}{m_N} h_{1L}^\perp)$$

PhH, B. Musch et al.
arXiv:0908.1283



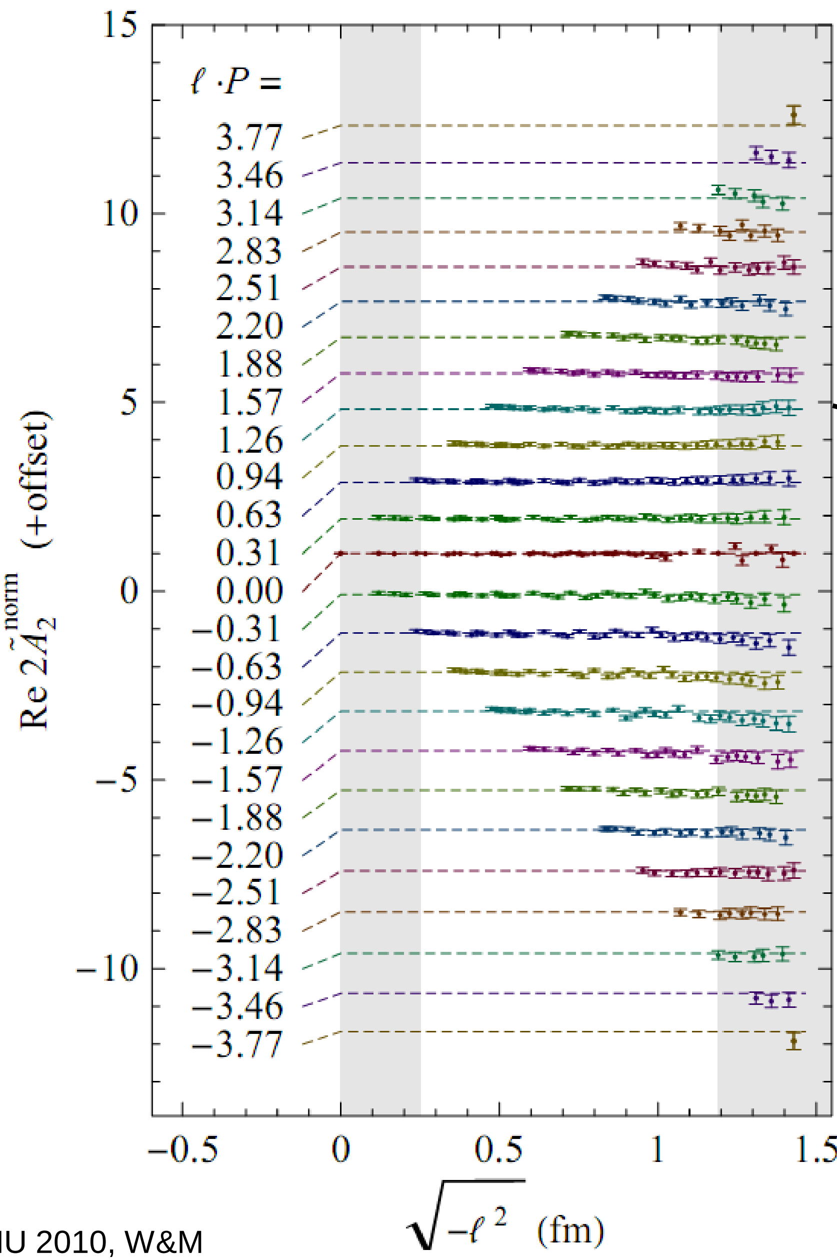
~~$h_{1L}^\perp(x, k_\perp) \leftrightarrow$ GPDs~~

genuine effect
of intrinsic transverse
momentum of quarks

Transverse momentum dependent PDFs

correlations in x and k_γ

Musch et al. n_f=2+1 mixed tbp and PoS LC2008



$$A_2^{norm}(l \cdot P, l^2) \equiv \frac{A_2(l \cdot P, l^2)}{A_2(l \cdot P = 0, l^2)}$$

no visible correlations in l ∢P and l^2

$k_\perp \leftrightarrow l_\perp$

$x \leftrightarrow l \cdot P$

≈ factorization of tmdPDFs in x and k_γ

$$f(x, k_\perp) \approx f(x) g(k_\perp)$$

Spin structure of the pion

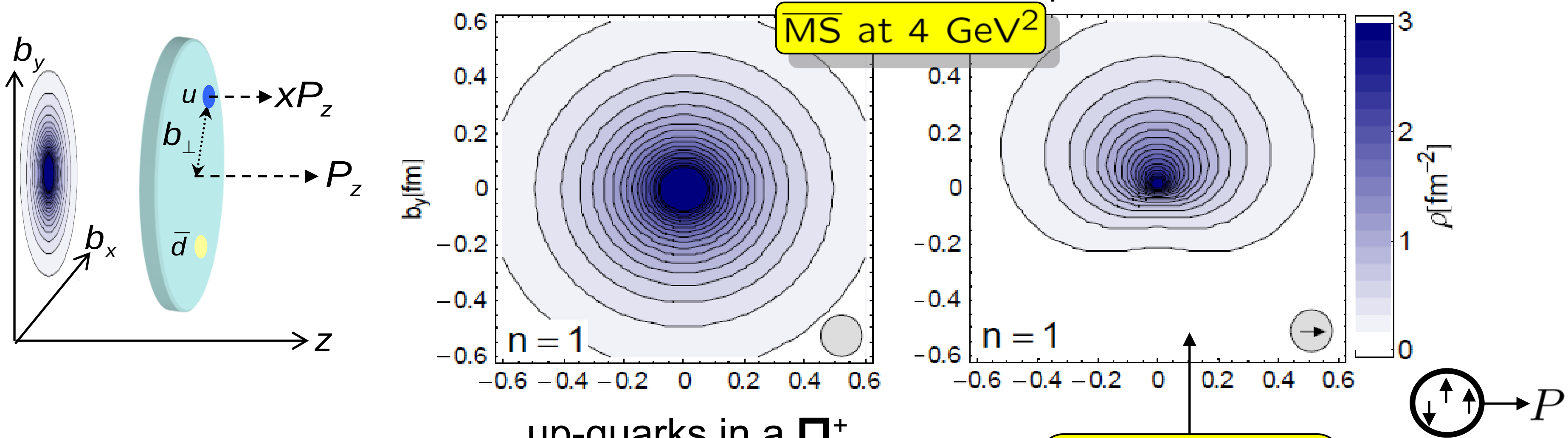
QCDSF n_f=2 Clover PRL 2008

$\pi^+ \widehat{=} u^\uparrow \bar{d}^\downarrow$ → pion spin sumrule → $0 = S_z = 0$ ↔ Is the pion spin structure trivial?

$$\rho_T(x, b_\perp; s_\perp) = \frac{1}{2}\Big\{H^\pi(x, b_\perp^2) - \epsilon_{ij} s_\perp^i b_\perp^i \frac{1}{m_\pi} E_T^{\pi\prime}(x, b_\perp^2)\Big\}$$

but is $E_T^{\pi\prime}$ non-zero?

quark transverse spin in x-direction



up-quarks in a π^+

QCDSF n_f=2 Clover, PRL 2008

lattice calculations of quark *spin-flip couplings*

the pion has a *non-trivial* transverse spin structure!